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The Emergence of Enforcement

Discussion Paper

SP II 2024–202

July 2024

WZB Berlin Social Science Center

Research Area
Markets and Choice

Research Unit
Market Behavior

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Abstract

The Emergence of Enforcement

by Luca Anderlini, Leonardo Felli, and Michele Piccione *

How do mechanisms that enforce cooperation emerge in a society where none are available and agents are endowed with just raw power that allows a more powerful agent to expropriate a less powerful one?
We study a model where expropriation is costly and agents can choose whether to engage in surplus-augmenting cooperation or engage in expropriation.
While in bilateral relations, if cooperation is not overwhelmingly productive and expropriation is not too costly, the latter will prevent cooperation, when there are three or more agents, powerful ones can become enforcers of cooperation for agents ranked below them. In equilibrium they will expropriate smaller amounts from multiple weaker cooperating agents who in turn will not deviate for fear of being expropriated more heavily because of their larger expropriation proceeds.
Surprisingly, the details of the power structure are irrelevant for the existence of equilibria with enforcement provided that enough agents are present and one is ranked above all others. These details are instead key to the existence of other highly noncooperative equilibria that are obtained in certain cases.

Keywords: Jungle, power structures, enforcement, rule of law

JEL classification C79, D00, D01, D31, K19, K40, K49.

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*E-mail: luca.anderlini@georgetown.edu, leonardo.felli@wzb.eu, m.piccione@lse.ac.uk. A previous version of this paper was entitled \The Emergence of the Rule of Law." We are grateful to Roger Lagunoff, Marco Pagano, Alan Schwartz, Sepehr Shahshahani, Josh Teitelbaum, seminar participants at EIEF, Federico II, University of Cambridge, and conference participants at LET XI Georgetown for very helpful comments. We also thank to Helen Watson for bibliographic references and Henrik Müller and Susanne Fengler for extremely helpful conversations. Leonardo Felli gratefully acknowledges support from the Keynes Fellowship at the University of Cambridge.

1. Introduction

Many studies have recently directed attention to the accumulation and storability of resources as key drivers for the emergence of hierarchies and the enforcement of property entitlements. This view, however, leaves one important question open: how does a society take the "first step" towards the enforcement of rule-based interactions? One can easily envisage a Hobbesian state of nature in which efforts directed at the accumulation of storable commodities are themselves frustrated by lawless appropriation. In this paper we investigate of how enforcement mechanisms may plausibly come about in a landscape in which they are not available to begin with.

We ask what seems *prima face* a difficult yet basic question. How do enforcers of property entitlements emerge when *none* are present to begin with? A classic view posits that the members of a society agree to a "social contract" and submit to the authority of selected individuals in exchange for maintaining a rule-based order. While such a view is appealing and has been hugely influential as a moral foundation of a social order,[1] it does not explain its actual origin.

In this paper we argue that, under plausible but by no means general conditions, in a society in which all relations are characterized only by raw power enforcement of property entitlements can emerge as an equilibrium phenomenon: some agents who seize wealth, because of their ability to do so, effectively act as enforcers by deterring other agents from using their own power to do the same.

The main feature of our primitives is a non-cooperative environment governed by force.[2] Piccione and Rubinstein (2007) term this environment the "jungle," and we do the same here. In the jungle, agents can use their power to seize the assets of other agents.[3] No other tools are available to regulate economic relationships. In particular, the possibility of any agreement that is not self-enforcing is precluded.

There are, of course, a variety of routes that could be followed when defining the "power" of agents over other agents. As in Piccione and Rubinstein (2007), we opt for a binary,

[1]Starting with the classical, the inevitable references are Plato (1943), Hobbes (1968), Locke (1952) and Rousseau (2004).

[2]We do not wish to pursue the analogy in any detailed way, but this is akin to the state of "anarchy" described by Nozick (1974). Notice, however, that the power structure in our analysis is key to the nonexistence of a stable anarchy as in Hirshleifer (1995). In a sense our analysis focuses on how society transitions from an anarchy to a stable structure based on enforcement.

[3]Throughout, we use the terms "seize," "appropriate," "expropriate," and "steal" interchangeably.

intransitive notion. When any two agents are compared either one is more powerful than the other and a strict ranking obtains, or the two are not comparable. An agent who is more powerful than another will be able (at a cost) to seize the assets of the latter.

In many jungles, two individuals in isolation will not be able to trade or undertake other surplus-creating activities. The more powerful of the two individuals will be able to expropriate the less powerful one and appropriate the proceeds as well as the initial assets of the other agent. Incentives to engage in surplus-creating cooperation are thus destroyed.[4]

In our jungle, expropriation is costly. Other things being equal, stealing more is more costly at an increasing rate so that an expropriating agent may not want to steal all available assets from less powerful ones. More importantly, stealing costs contain a *strategic component*: stealing from less powerful agents who have used resources for expropriating other agents is comparatively cheaper. Thus, stealing may entail a strategic disadvantage for an agent: accumulating stolen wealth makes him a more desirable target for expropriation by more powerful agents.

Based on these two ingredients, the simplest version of our main result runs as follows. While two agents fail to trade in isolation as the more powerful will expropriate the less powerful one, they may in fact succeed when a *third* agent more powerful than both is present. In particular, the third agent expropriates *both agents but each by a smaller amount*, thus giving them "enough room" to generate the potential surplus that they can achieve. Indeed, the more powerful agent in the original pair does not find it profitable to expropriate the other since he would face more expropriation by the third agent as the latter would now find it less costly to extract resources from him. Thus, the third agent effectively acts as an "enforcer" of the trade, or other surplus-creating relationships. In this sense, enforcement emerges endogenously from the jungle as an equilibrium phenomenon.

We say that equilibria exhibit endogenous enforcement when some agents refrain from using their power to expropriate the resources of other agents. Surprisingly, endogenous enforcement generalizes considerably with respect to the power structure that characterizes the jungle. Such equilibria are present not only when agents are ranked in a "linear order," but also in more general structures in which power is only a partial order of the agents.

Somewhat counterintuitively, while the details of the power structure are irrelevant for the

[4] This, of course, is far from universally true. For instance, stealing could be a very costly activity and/or the surplus generated could be large enough that all involved would find it profitable to participate.

emergence of enforcement provided that the jungle is inhabited by a sufficiently large number of agents, they are instead relevant for the existence of equilibria in which expropriation is pervasive. Roughly speaking, more "fanning out" in the power structure (as opposed to a linear order) corresponds to the possibility of highly non-cooperative equilibria in which no trading opportunities are realized and only expropriation occurs.

2. Related Literature

We have mentioned already that we borrow some of the basic ingredients from the jungle of Piccione and Rubinstein (2007).[5] It should be noted that in their jungle there are no expropriation costs and our results instead revolve around their presence.

A point to clear up first is that we do *not* pursue explanations of enforcement based on long-run relationships (repeated or open-ended). We do not attempt to present here an exhaustive list of references on relational contracts. For the more game-theoretic side we refer to Mailath and Samuelson (2006), while for the more contract-theory angle we rely on the surveys by Gibbons and Roberts (2013) and Malcomson (2013).[6] Our model is a stylized two-period game. We deliberately do not rely on repeated-game interactions.[7]

A separate legal literature examines "framework agreements" in which strong relational long-run aspects are focal, asks what forms of enforcement are available in these cases, and finds that these are lacking in some important respects. In a recent paper Schwartz and Sepe (2022) propose a novel legal structure for the tools of enforcement to cover some of these gaps.

The effects of the availability of enforcement and the type of legal system it is embedded in have also been the subject of extensive studies. Acemoglu and Johnson (2005) study the role of enforcement stemming from property rights and their historical role on a global basis. Glaeser and Shleifer (2002) examine the role of legal origins — "how" enforcement is deployed, common versus civil law — on economic and social outcomes. These literature streams have concentrated on the role of enforcement mechanisms and their origins on subsequent outcomes.[8] In a sense, here we go one step further upstream and ask how enforcement mechanisms may have emerged in the first place in a landscape where none were available.

[5]See also Piccione and Rubinstein (2004) and the more recent Rubinstein and Yildiz (2022).

[6]See also the recent Calzolari, Felli, Koenen, Spagnolo, and Stahl (2021) for a specific empirical case study.

[7]In this respect our setting also differs from the one in Acemoglu and Wolinsky (2020) where the individual enforcers are disciplined through a mechanism sustained by repeated social interactions.

[8]See also Anderlini, Felli, and Riboni (2020).

In an earlier contribution, Moselle and Polak (2001) examine a model of a predatory state in which no equilibria with enforcement arise. In their model the key strategic component of the stealing costs is notably absent and this is one of the elements, in addition to the obvious differences in the strategic setup, that drive the differences between their findings and ours.

A series of studies examine the Sicilian Mafia as an example of "privately provided" enforcement. This literature takes off with a sociological study by Gambetta (1996) documenting how the private provision of protection and enforcement is the main business of the Mafia. Bandiera (2003) models and tests privately provided enforcement it in a common-agency setting. In a sequence of papers Dixit (2003a,b, 2009, 2011) investigates theoretical models of private provision of protection and enforcement and their effects on economic governance.

Our contribution is related to some of the recent literature on the "neolithic revolution:" the transition from hunter-gatherers to agriculture that eventually led to complex hierarchical states, complete with enforcement mechanisms including the ones that enable tax-levying. This literature is vast and we do not even attempt to survey it here. The dominant view is that following the adoption of farming the newfound surplus of storable food and the need to allocate it somehow led to complex hierarchical societies.

One empirical study closely related to our theoretical model is a recent paper by Mayshar, Moav, and Pascali (2022) to which we will return shortly. One of their main points of departure is in turn well expounded by Scott (2017). The key observation is that *not all* types of agricultural crops led to the same outcomes. In particular Scott (2017) writes:

> *It is surely striking that virtually all classical states were based on grain* [...] *History records no cassava states, no sago, yam, taro, plantain, breadfruit, or sweet potato states.*

The difference between grains and the rest is that they are *storable* and easier to transport because of their lower water content. The upshot of the empirical tour de force by Mayshar, Moav, and Pascali (2022) is that:

> *In summary, our empirical analysis provides repeated evidence that the cultivation of cereals had a significant causal effect on the development of complex hierarchies and states, consistent with the appropriability theory. It also shows that the correlation between land productivity and hierarchy disappears when the cultivation*

> *of cereals is controlled for, consistent with our critique of the conventional productivity theory. Moreover, the finding that it is unlikely that complex hierarchies would emerge when productive roots and tubers are available supports both the appropriability theory and the critique of the productivity theory.*

In a related earlier study, that takes the emergence of elites as given, Mayshar, Moav, and Neeman (2017) examine the effect of "transparency" and hence appropriability of revenue from crops on the shape of institutions. Among other things, their detailed empirical work focuses on the differences between ancient Egypt and southern and northern Mesopotamia.

The distinction between appropriable, storable grains and other crops has a notable incarnation in our model both in the *strategic component* of the cost of stealing and in the possibility that stolen wealth flows upstream via appropriations.

3. Simple Environments

We begin by illustrating with a simple numerical example how the emergence of enforcement that we investigate in this paper can take place.

Consider a jungle populated by two agents who find it impossible to cooperate and create surplus because one is more powerful than the other. Surplus creation does not take place because the more powerful agent will find it more profitable to expropriate the less powerful (in fact powerless in this case) agent than to engage in surplus-creating cooperation.

The relationship between these two agents changes dramatically when we add a third agent to the jungle more powerful than both the original ones. The more powerful agent in the two-agent set-up now becomes the intermediate agent. In equilibrium the more powerful agent expropriates both the original agents, but by a smaller amount because of expropriation costs. The intermediate agent refrains from expropriating the powerless agent and hence the surplus-creating activity can be sustained. The intermediate agent refrains from expropriation because if he did not the more powerful agent would find it to his advantage to expropriate only the intermediate agent by a larger amount since this will save on his expropriation costs.

3.1. A Jungle With Two Agents

To avoid having to rename them later, the two agents in the initial case are named $i = 2, 3$. Agent 2 is more powerful than agent 3, written as $2 \succ 3$. At a cost, 2 can expropriate 3.

In the absence of surplus-creating cooperation, the endowments of the two agents are $\omega_2 = \omega_3 = 100$. Cooperation enhances the endowments: they become $\tilde{\omega}_2 = \tilde{\omega}_3 = 110$.

Throughout the paper, we consider a framework with two time periods, $t = 0$ and $t = 1$. At $t = 0$ the agents, both 2 and 3, decide simultaneously and independently whether to cooperate or not.[9] Once taken, the decisions to cooperate or not become observable to all. The choice not to cooperate can be interpreted as undertaking an (observable) activity that will enable an agent to engage in costly expropriation (for instance investing in offensive hardware). An agent who chooses to cooperate cannot later on (at $t = 1$) engage in costly expropriation — he lacks the "offensive means" to do so.

If both agents decide to cooperate then surplus grows and no expropriation takes place at $t = 1$. So, in this case the agents both receive payoff $\tilde{\omega}_i = 110$ and nothing further takes place. If instead at $t = 0$ at least one of the two agents decides not to cooperate then we enter $t = 1$ with the agents having their original endowments $\omega_i = 100$. If 2 has decided not to cooperate he will then be able to expropriate at a cost (part or all of) the endowment of the weaker agent 3.

Anticipating the notation we will use below, we let the amount by which 2 expropriates 3 be denoted by s_{23}. In this example we assume that the cost of expropriation that 2 incurs as a function of s_{23} is such that the optimal stolen amount is 80,[10]. The expropriation cost that 2 incurs when stealing the endowment of 3 is $\mathcal{C}(s_{23}) = 40$.

It now follows easily that 2 will decide not to cooperate at $t = 0$ and then proceed to expropriate 3 at $t = 1$. Indeed, if he cooperates he will end up with a payoff of at most $\tilde{\omega}_2 = 110$. If instead he decides not to cooperate he will end up with a payoff of $\pi_2 = \omega_2 + s_{23} - \mathcal{C}(s_{23}) = 100 + 80 - 40 = 140$.[11]

In this two-agent jungle even though surplus-augmenting cooperation is possible in principle it will not take place because of the expropriation power that 2 holds over 3.

[9] As will be apparent, the fact that 3 has something to decide is completely immaterial here. In fact it only generates the possibility that cooperation may not arise because of an equilibrium "coordination failure" between 2 and 3. To avoid this "spurious" possibility, in the full-fledged model we describe in Section 4 (Subsection 4.2) below powerless players (like 3 in this example) will be assumed to take no decision at all.

[10] All the quantities we use here and below in Subsection 3.2 are consistent with the actual cost function we describe in Section 5. See footnote 18 below where this is made explicit.

[11] Notice that the fact that 2 decides to expropriate is not true for all numerical values. However here the *net* benefit to 2 from expropriation is equal to $80 - 40 = 40$, which is greater than the up-front benefit from cooperation, namely $110 - 100 = 10$. To make the problem interesting, we will always assume that this is the case. This is the purpose of Assumption 2 below.

3.2. A Jungle With Three Agents — The Emergence of Enforcement

We now add a third agent to the jungle, agent 1, who is more powerful than both 2 and 3. So, we now have a jungle populated by $1 \succ 2 \succ 3$.

The endowments of 2 and 3 in case of cooperation or non cooperation are the same as before (namely 110 and 100 respectively). The simplest way to proceed to make the basic point is to assume that 1's endowment is not affected by his decision to cooperate or not. Hence, we just set $\omega_1 = \tilde{\omega}_1$, the value of which is immaterial to the rest of our example.

The effect of $t = 0$ cooperation decisions on endowments is just as in Subsection 3.1 above. If both 2 and 3 decide to cooperate their endowments grow from 100 to 110. If either decides not to cooperate both of their endowments remain at 100.

Expropriation costs are as in Subsection 3.1. So, the optimal amount that a non-cooperating agent expropriates from less powerful agents who have not stolen from agents below them remains at 80, and the expropriation cost also stays at 40.

There are three additional features of the expropriation costs that come into play in the three-agent jungle. The first is that if an agent is expropriating multiple agents below him who have accumulated the same expropriated amount, he is indifferent as to whom he in fact expropriates since the cost is the same. So, for instance 1 could (as will be the case in the equilibrium we construct) expropriate 40 each from 2 and 3, and incur a total cost of 40. Second, agents who have accumulated more expropriated resources are comparatively cheaper to expropriate. With the expropriation costs we have in mind (see footnote 10) an agent who expropriates from an agent below who in turn has stolen a total of 80 will find it optimal to steal 120 and incur a cost of 60.

Our candidate equilibrium is that at $t = 0$ agents 2 and 3 decide to cooperate while 1 decides not to cooperate. At $t = 1$, agent 1 then expropriates agents 2 and 3 in equal amounts adding up to 80 so that $s_{12} = s_{13} = 40$ at a total cost of 40. In this candidate equilibrium 1 ends up with a payoff of $\omega_1 + 80 - 40$. Agents 2 and 3 both end up with a payoff equal to their cooperating endowment minus what agent 1 expropriates from them, namely $110 - 40 = 70$.

Checking for deviations by 1 is trivial. The only one to consider is that he could decide to cooperate at $t = 0$ and forfeit his ability to expropriate at $t = 1$. If 1 deviates in this way he ends up with a payoff simply equal to his endowment, namely $\tilde{\omega}_1 = \omega_1$. This is a net loss of 40 relative to the proposed equilibrium.

Checking for deviations by agent 3 is also simple enough. Whether he decides to cooperate or not, he is a powerless agent and cannot expropriate anyone in any case. So, if at $t = 0$ he were to deviate and decide not to cooperate, his endowment would stay at 100 and 40 of this endowment would still be expropriated by 1. So, he would end up with a payoff of 60, with a net loss of 10 relative to the proposed equilibrium.

Next, consider agent 2. If at $t = 0$ he decided not to cooperate, his optimal decision at $t = 1$ would be to expropriate 80 from agent 3 at a cost of 40. However, following the decisions of both agents 1 and 2 not to cooperate the behavior of agent 1 would also change at $t = 1$.[12]

As we mentioned above, we assume that the cost of expropriation is such that it is preferable (less costly) to expropriate an agent who has accumulated more expropriated wealth from others. Viewed from the position of agent 1, agent 2 now has a net expropriation balance of 80 while the powerless 3 of course has balance of -80. Hence, with the expropriation cost function we have in mind (see footnote 10 again) after the deviation by 2, agent 1 will find it optimal to expropriate 120 from 2 alone at a cost of 60. Hence following a deviation to not cooperate at $t = 0$, agent 2 ends up with a payoff composed as follows. His endowment is $\omega_2 = 100$, he expropriates $s_{23} = 80$ from agent 3 at a cost of $\mathcal{C}(s_{23}) = 40$, and he is expropriated $s_{12} = 120$ by agent 1. Hence his overall payoff is $100 + 80 - 120 - 40 = 20$, a net loss of 50 relative to the proposed equilibrium.

Recall that in the jungle populated by only agents 2 and 3, no surplus-generating cooperation was possible in equilibrium. This is the reason why we interpret the equilibrium in which agents 2 and 3 engage in surplus-generating cooperation while getting expropriated by 1 as a situation in which 1 plays the role of *enforcer of the cooperation* between 2 and 3.

4. A General Environment

4.1. Agents and Power Structures

There is a set $\mathcal{P} = \{1, ..., N\}$ of agents, with $N \geq 3$. The power structure according to which these agents are arranged is one of the key primitives of the model. As we mentioned we opt for a binary notion of power for which when two agents are compared either one is more powerful than the other, so that a strict ranking obtains, or the two are not comparable. An

[12] Play would be in a different $t = 1$ subgame.

agent who has power over another agent will be able (at a cost) to seize resources belonging to the other agent.

Formally, we have a binary relation $\succ$ that is a *strict partial order* on $\mathcal{P}$, so that $\succ$ is irreflexive and transitive. In addition $\succ$ must satisfy two conditions. First, $1 \succ i$ for every $i \neq 1$. Second, for any $i, j, h \in \mathcal{P}$ such that $j \succ i$ and $h \succ i$, either $j \succ h$ or $h \succ j$, so that all players have exactly one immediate predecessor. A pair $(\mathcal{P}, \succ)$ that satisfies the above conditions is known as an *arborescence with a root* (in our case player 1).[13]

The set of players j such that $i \succ j$ is denoted by $\mathcal{B}_i$ and the set of players j such that $j \succ i$ is denoted by $\mathcal{A}_i$. We will often refer to players in $\mathcal{B}_i$ and $\mathcal{A}_i$ as players "below" and "above" i respectively. If for some i we have that $\mathcal{B}_i = \emptyset$ we say that i is a "powerless" player. We will rule out the case where all players but player 1 are powerless. The following assumption will hold throughout.

Assumption 1. *General Power Structures: The players $\mathcal{P}$ together with the binary relationship $\succ$ form an arborescence with a root, namely player 1. Furthermore, $\mathcal{B}_i \neq \emptyset$ for some $i \neq 1$.*

We refer to a pair $(\mathcal{P}, \succ)$ satisfying Assumption 1 as a *General Power Structure*. In Section A.1 in the Appendix we report a few properties of arborescences and their "subsets" known as sub-arborescences that prove to be useful in our analysis.

4.2. Endowments, Timing, Cooperation and Expropriation Costs

The players make decisions in two periods, $t = 0, 1$. In the first period $t = 0$, each player i is endowed with an amount of resources ω_i and decides whether to "cooperate" or not to cooperate and thus "activate" his power to expropriate players below him. The decision by player i to cooperate is simply an irreversible commitment[14] *not* to subsequently expropriate any of the players in $\mathcal{B}_i$. If player i is powerless such commitment is inevitable and thus any powerless player will always be assumed to cooperate. Cooperation decisions are taken

[13] We return to the role of having a single most-powerful agent in Section 8 that concludes the paper.

[14] We do not use the term "cooperation" in a literal sense. The decision to cooperate or not is taken (and has effect) *unilaterally* by each player, without the need to other players to cooperate. This is simply to avoid a multiplicity of equilibria that would be generated by potential "coordination failures," where players do not cooperate simply because they do not expect their potential partners to cooperate (see also footnote 9 above). While such multiplicity would add to the richness of the model, on balance we think it would be mainly a distraction from the main narrative we are pursuing here.

simultaneously and independently at $t = 0$ and once taken are observable by all players. Hence a set of $t = 0$ decisions to cooperate or not by all players identifies a $t = 1$ *subgame.*

The decisions to cooperate or activate one's power could be viewed as observable choices to engage in time consuming activities that preclude or enable the use of a player's power to expropriate other players. For instance, some players may plant wheat and other acquire offensive weapons that enable expropriation.

In the second period, $t = 1$, if a set $\mathcal{D}$ of players has chosen to cooperate in the first period their available resources *grow* while the resources of the players who do not cooperate remain constant. The state of player i's resources at the beginning of the second period is denoted by $\omega_i^{\mathcal{D}}$ where $\omega_i^{\mathcal{D}} = \tilde{\omega}_i > \omega_i$ if $i \in \mathcal{D}$, and $\omega_i^{\mathcal{D}} = \omega_i$ if $i \notin \mathcal{D}$. Often, it will also be convenient to refer directly to the set of non-cooperating players $\mathcal{Z} = \mathcal{P}/\mathcal{D}$.[15]

In the second period, after the set $\mathcal{D}$ and all the $\omega_i^{\mathcal{D}}$ are realized, the expropriation decisions are taken. Suppose that i is in $\mathcal{Z}$. We denote by s_i the vector with elements s_{ij} each indicating the amount that player i chooses to expropriate from $j \in \mathcal{B}_i$.[16] If i instead is in $\mathcal{D}$ then s_i is a vector of zeroes of length $\|\mathcal{B}_i\|$. Let $\boldsymbol{s}$ be the array $\{s_i\}_{i=1}^N$ that describes all expropriation decisions as constrained by the general power structure $(\mathcal{P}, \succ)$ and by the cooperation decisions. As standard, we will denote by s_{-i} the array $\boldsymbol{s}$ with the vector s_i removed. Any non-cooperating player i takes s_{-i} as given when making his own decision about s_i.

Remark 1. *Ranking of Cooperating Players: Notice that we are positing that a power structure is unaltered by the decision to cooperate. Cooperating players cannot be expropriated by players that are ranked below them and who have activated their power. This modeling choice may be more or less appealing, depending on how "hard-wired" into the players one wishes to interpret to be the notion of power.*

We proceed in this way since it simplifies the notation and the analysis. With the exception of Proposition 5, our results remain valid if all players who choose to cooperate join one of their powerless successors in the power ranking while the players who activate their power maintain their rank according to the original power structure.

Expropriation is costly. When a non-cooperating player i selects a vector of expropriations s_i, she will bear an expropriation cost denoted by $\mathcal{C}_i(s_i, s_{-i})$ where the dependence on both

[15] All powerless players are assumed to be automatically in $\mathcal{D}$. See footnote 9 above.

[16] The quantities s_{ij} need to satisfy certain inequality constraints that will be specified shortly.

s_i and all other players expropriation choices s_{-i} is made explicit.

Given the array $\boldsymbol{s} = (s_i, s_{-i})$ of expropriation decisions, the payoff of a non-cooperating player i is equal to her net final resources obtained and surrendered via expropriation, minus the expropriation costs:

$$\pi_i = \omega_i + \sum_{j \in \mathcal{B}_i} s_{ij} - \sum_{q \in \mathcal{A}_i} s_{qi} - \mathcal{C}_i(s_i, s_{-i}) \tag{1}$$

If player i is cooperating, she will enjoy a payoff equal to her resources from cooperation minus the amount that is expropriated from her:[17]

$$\pi_i = \tilde{\omega}_i - \sum_{q \in \mathcal{A}_i} s_{qi} \tag{2}$$

5. Expropriation Costs

5.1. Preliminaries

Consider the second period $t = 1$, after all cooperating decisions have been taken and observed by all players, and a non-cooperating player i. Since such player takes s_{-i} as given and his payoff is as in (1) above, the choice problem of i is to select a vector s_i so as to maximize

$$\sum_{j \in \mathcal{B}_i} s_{ij} - \mathcal{C}_i(s_i, s_{-i}) \tag{3}$$

We are now ready to define the class of cost functions that we will work with throughout the paper. Consider a pair (i, j) with $j \in \mathcal{B}_i$ and let

$$h_j = \sum_{z \in \mathcal{B}_j} s_{jz} \tag{4}$$

so that h_j are the expropriation *gains* to player $j \in \mathcal{B}_i$ that player i takes as given.

Player i's expropriation costs are of the form

$$\mathcal{C}(s_i, s_{-i}) = \mathcal{C}^1 \left[\sum_{j \in \mathcal{B}_i} s_{ij} \right] + \sum_{j \in \mathcal{B}_i} s_{ij}\, \mathcal{C}^2(-h_j) \tag{5}$$

[17] The amounts expropriated need to be consistent since no player can be left with a negative amount of resources. We return to this issue below in Subsection 5.2.

where $\mathcal{C}^1(\cdot)$ and $\mathcal{C}^2(\cdot)$ are strictly increasing, $\mathcal{C}^1$ is strictly convex and satisfies standard Inada conditions, $\mathcal{C}^1(0) = 0$, and $\mathcal{C}^2(\cdot) \geq 0$.

The first term is a standard "effort" cost which depends only on the total amount that player i steals. The second term in (5) is the strategic crowding out term that we have referred to multiple times above. The less resources player j expropriates from other players, the higher the *marginal* cost of expropriating player j. Players who have accumulated more resources though expropriation are more vulnerable to attack from non-cooperating players above them.

In our view, to properly interpret the strategic crowding out term in our cost of stealing function requires that the resources that are the subject of expropriation are storable and hence somewhat durable as well. If it is easier to steal from a player who has a higher expropriation surplus, presumably this is because the resources to be stolen are (at least in part) those that he has actually stolen from others. It is almost tautological that since the cost of stealing from a player depends on how much he has accumulated in stealing from others, the resources must be of an "accumulable" nature. As we noted above, this is a point that figures prominently in the findings of Mayshar, Moav, and Pascali (2022) that we discussed at length in Section 2 above.

A natural observation at this point would be to say that a player that has accumulated more resources in stealing could use them to increase his defense capabilities and hence make it more costly for other players to steal from him. We do not explicitly consider this effect since we view it as less "primitive" than the "liquidity" effect that we emphasize here that has the opposite sign. This is because our main purpose is precisely to explain the emergence of a social structure that may enable such complex and stratified defense strategies in the first place.

Before moving on, it is helpful to point out that a "canonical" class of cost functions that fits (5) is

$$\mathcal{C}(s_i, s_{-i}) \;=\; A\left[\sum_{j\in\mathcal{B}_i} s_{ij}\right]^{\gamma} \;+\; B\sum_{j\in\mathcal{B}_i} s_{ij}(\hat{h} - h_j) \tag{6}$$

where γ, $\hat{h}$, A and B are constants that satisfy $\gamma > 1$, $A > 0$ and $0 < B < 1/\hat{h}$. The numerical examples that we discussed in Section 3 were constructed using this class.[18]

[18] In particular, the values we used in the numerical examples involving two and three agents in Sections

5.2. *Rationing and Interiority*

It is clear that the expropriation vectors in the subgames that follow the first period decisions could be inconsistent if the total amount expropriated from a player j could exceed the player's resources.

A possible way to resolve this inconsistency would be to include rationing rules that constrain the actual amounts that are expropriated in such cases. Following this route, each s_{ij} can be viewed as the effort of player i intended for expropriating s_{ij} from player j, which, depending on the effort allocated by other players and the resources of player j, may or may not result in player i expropriating s_{ij} units of resources. The attempt of player i to expropriate j may succeed or only partially succeed.

Remark 2. *Rationing Excess Expropriation: If we pursued the route of rationing rules to ensure consistency of the expropriation "requests" or "efforts," the payoff of a non-cooperating and a cooperating player would be respectively defined as*

$$\omega_i + \sum_{j \in \mathcal{B}_i} \alpha_{ij}(\boldsymbol{s}) - \sum_{q \in \mathcal{A}_i} \alpha_{qi}(\boldsymbol{s}) - \mathcal{C}_i(s_i, \alpha_{-i}(\boldsymbol{s})) \tag{7}$$

and

$$\tilde{\omega}_i - \sum_{q \in \mathcal{A}_i} \alpha_{qi}(\boldsymbol{s}) \tag{8}$$

where each $\alpha_{ij}(\boldsymbol{s}) \leq s_{ij}$ satisfies suitable (but inevitably somewhat arbitrary) constraints to ensure aggregate consistency of the expropriation outcomes.

In what follows, we choose to pursue a straightforward alternative to arbitrary rationing rules which also simplifies the analysis and in our view makes the results considerably more transparent. We assume that, given the cost of stealing, the players' endowments are sufficiently large so as to guarantee that the expropriation choices are mutually compatible in the sense that the constraint that no player is left with negative resources is simply not binding. In other words the expropriation choices are "interior" since endowments are large enough. Introducing rationing rules that satisfy a mild set of "monotonicity" restrictions would leave the qualitative nature of our results unaffected.

3.1 and 3.2 can be recovered using the cost function in (6) and setting $A = B = 1/320$, $\hat{h} = 160$ and $\gamma = 2$.

We relegate the formalities of the interiority of solutions when endowments are large enough to Section A.2 in the Appendix. The formalities require a surprising amount of detail, and, in our view, they would just constitute a distraction if presented here. The following remark suffices to clarify.

Remark 3. *Large Endowments and Interiority: Fix two functions $\mathcal{C}^1(\cdot)$ and $\mathcal{C}^2(\cdot)$ as above. We can then find a lower bound $\underline{\omega}$ on endowments such that if $\omega_i \geq \underline{\omega}$ for every i the following is true.*[19]

Consider any set of $t = 0$ cooperating choices. Let $\mathcal{D}$ be the set of cooperating players, and consider any $i \notin \mathcal{D}$. Consider any set of "undominated" expropriation choices s_{-i} for all players except for i.[20] *Then the optimal expropriation choices s_i for i are consistent with the arbitrary choices s_{-i} in the sense that they do not leave any player with a negative amount of resources.*[21]

For the remainder of the paper, we assume that endowments are sufficiently large in the sense we have just specified.

5.3. The Problem Of Player i

We now sum up again the problem that any non-cooperating player i faces in the second stage of the game. Using (5), this is given by[22]

$$\max_{s_i=\{s_{ij}\}_{j\in\mathcal{B}_i}} \quad \sum_{j\in\mathcal{B}_i} s_{ij} - \mathcal{C}^1\left[\sum_{j\in\mathcal{B}_i} s_{ij}\right] - \sum_{j\in\mathcal{B}_i} s_{ij}\, \mathcal{C}^2(-h_j) \tag{9}$$

where h_j is as in (4) above.

In general Problem (9) admits multiple solutions. However, it is easy to see that the solution is unique if there are no "ties" among the values of the h_j. Proposition 1 below fully

[19] What follows is an informal restatement of Proposition A.1.
[20] See Definition A.3.
[21] See Subsection 5.3 below.
[22] We are ignoring the constraints that stipulate that expropriation choices should not leave any player with negative resources. We do this on the basis of our assumption of sufficiently large endowments. See Remark 3 and the discussion in Subsection 5.2 as well as Section A.2 in the Appendix.

characterizes the solutions. Define

$$\overline{h}_i \;=\; \max_{q\in\mathcal{B}_i}\{h_q\} \quad \text{and} \quad \mathcal{M}_i = \{q \in \mathcal{B}_i \;|\; h_q \;=\; \overline{h}_i\} \tag{10}$$

so that $\overline{h}_i$ is the maximal value of h_q among players $q \in \mathcal{B}_i$, and $\mathcal{M}_i$ is the set of players in $\mathcal{B}_i$ who have such maxim value. Hence $\mathcal{M}_i$ is the set of players below i who have the highest accumulated stolen wealth, that i takes as given. Given any h define also

$$z(h) \;=\; \operatorname*{argmax}_x \,\{x \;-\; \mathcal{C}^1(x) \;-\; x\,\mathcal{C}^2(-h)\} \tag{11}$$

The behavior of a non-cooperating player i in period $t = 1$ is then characterized by his best response to s_{-i} which is the object of Proposition 1 that follows.

Proposition 1. *Best Response: Let $s_i^* = \{s_{ij}^*\}_{j\in\mathcal{B}_i}$ be a solution to Problem (9). Then the quantities s_{ij}^* are fully characterized by the following conditions.*[23]

(i) The total quantity stolen $\sum_{j\in\mathcal{B}_i} s_{ij}^*$ by player i is equal to $z(\overline{h}_i)$.

(ii) If $j \notin \mathcal{M}_i$ then $s_{ij}^* = 0$

(iii) $\sum_{j\in\mathcal{M}_i} s_{ij}^* = z(\overline{h}_i)$

Proof: See Section A.3 in the Appendix.

The content of Proposition 1 is easy to summarize intuitively. A non cooperating player i will best respond to s_{-i} in period $t = 1$ as follows. He will only steal from those players who have the maximum stolen wealth among the players in $\mathcal{B}_i$. If there are two or more players that have the maximum balance i will then be indifferent as to whom he steals from. Only in this case i will have multiple best responses. The total amount that i steals from players in $\mathcal{B}_i$ is determined once we know the actual maximum stolen wealth among players in $\mathcal{B}_i$ and is increasing in this amount.

5.4. Gains From Stealing and Cooperation

Consider the extreme example of a player i who does not cooperate when all others do. By doing so, he will forgo the gain from cooperating, namely $\tilde{\omega}_i - \omega_i$, and he will gain the amount $z(0)$ specified in (11). If the gain from expropriation does not exceed the direct loss from

[23] Whenever $\|\mathcal{M}_i\| = 1$ we also know that the solution to Problem (9) is unique.

lack of cooperation, we are clearly in a case that is not particularly interesting. Even under the most conducive circumstances, player i will not choose to expropriate players below him. Our next assumption rules out that this is the case for any i.

Assumption 2. *Gains from Expropriation and Cooperation Condition: For every player i we have that*

$$z(0) - \mathcal{C}^1(z(0)) - z(0)\,\mathcal{C}^2(0) > \tilde{\omega}_i - \omega_i \tag{12}$$

We assume that this is the case for the remainder of the paper. Notice that if Assumption 2 were violated we would be in the case that Mayshar, Moav, and Pascali (2022) refer to as the "curse of plenty." The low-hanging fruit from cooperation is so plentiful that (costly) expropriation simply does not take place.

For completeness, before proceeding we state the following two remarks without proof.

Remark 4. *No Stealing Equilibrium: Suppose that (12) is violated for every player i. Then there is an equilibrium in which all players choose to cooperate at $t = 0$ and hence no stealing takes place at $t = 1$.*

Suppose next that Assumption 2 holds but, contrary to what we are actually assuming, the $\mathcal{C}^2(\cdot)$ component of the stealing costs in (5) is identically equal to zero. This is the case in which the strategic component of stealing costs does not exist. Notice that in this case, using Proposition 1, any player who decides not to cooperate will steal a total amount equal to $z(0)$ from his successors. The following remark shows why this case is not very interesting. Practically any stealing pattern can be bootstrapped to be an equilibrium.

Remark 5. *Bootstrapped Equilibria: Suppose the strategic component of the stealing costs is absent. Consider an arbitrary profile of first-period actions that stipulates for each non-powerless player except player 1 whether he cooperates or not. Furthermore, for each player that does not cooperate the strategy profile stipulates an arbitrary division of $z(0)$ to be appropriated from his powerless successors. The profile also stipulates that 1 does not cooperate and steals $z(0)$ from an arbitrarily chosen powerless player. Any deviation in the first period is punished by player 1 appropriating an arbitrary division of $z(0)$ only from the deviating players.*

It can be easily shown that this profile is a subgame perfect equilibrium.

6. General Power Structures

As we have mentioned before, our main result consists in identifying an equilibrium of the model that can be interpreted as one in which cooperation is centrally enforced.

This result holds, in our view surprisingly, for any power structure that satisfies Assumption 1, provided that Assumption 2 also holds and that N is large enough.

A preliminary concern regards the possible use of mixed strategies. However it would seem odd that to model the endogenous emergence of enforcement, randomized actions were essential. In society the actions of enforcers are clearly subject to noise. The noise, however, naturally stems from informational imperfections and actual mistakes of various kinds. Our model is stylized in the extreme so that mistakes and informational imperfections are ruled out. Given these premises, to home in on an enforcement arrangement that is intrinsically random would seem counterintuitive.

Our first result asserts that pure strategy equilibria exist and hence recourse to mixed strategies is unnecessary.

Proposition 2. *Existence of Pure Strategy Equilibria: Consider a General Power Structure as in Assumption 1 and let Assumption 2 hold. Then the game has a subgame perfect equilibrium in pure strategies.*

Proof: See Sections A.4 and A.5 in the Appendix.

In view of our remarks above, and of Proposition 2, throughout the rest of the paper we focus exclusively on pure strategy equilibria.

We are now ready to state formally our main result. If N is large enough our model has an equilibrium with "Centrally Enforced Cooperation."

Proposition 3. *Equilibrium with Centrally Enforced Cooperation: Consider a General Power Structure as in Assumption 1 and let Assumption 2 hold.*

Then there exists an $\underline{N}$ such that whenever $N \geq \underline{N}$ the game has an equilibrium in which player 1 does not cooperate while all other players cooperate. The lower bound $\underline{N}$ does not depend on the power structure of the players.

Proof: See Section A.6 in the Appendix.

One of the appeals of Proposition 3 is that, remarkably in our view, the result does *not depend* on the power structure at hand. Arborescences come in many "varieties." For instance $\succ$ could be a complete (or linear) order, so that the arborescence does not "fan out" at all. On the other hand the arborescence could at various points fan out with one player having a relatively large number of immediate successors. This in turn could happen at many points in the arborescence, or just a few. Branching could be symmetric or highly asymmetric.

Proposition 3 does not say much beyond the existence of the particular equilibrium it singles out. As it turns out the other equilibria that may be available and their features *do depend* on the features of the arborescence at hand, and on the class of equilibria that one chooses to focus on. This is what we investigate in Section 7 below.

7. The Structure of Power Structures

While the details of the power structure are irrelevant for the existence of equilibria with centrally enforced cooperation, they are important for the existence of equilibria in which instead many players do not cooperate. Our purpose in this section is to partially characterize how this is so. The main line of argument is in three steps.

First we define a property of equilibria that is interesting in its own right and show that all equilibria of the model display it. This property is key in making the problem tractable. Second, we introduce a key intermediate result (Proposition 5) about the possibility of equilibrium non-cooperation. Third, using the preliminary result we bring out what in our view is the "main wisdom" to be gained here.

The punchline is that when the details of the power structure comprise significant fanning out with one or more players having many direct successors then highly non-cooperative equilibria emerge, while when this is not the case the number of non-cooperating players in equilibrium has a very tight upper bound. We illustrate this punchline in Corollaries 1 and 3 below in which we focus on the two canonical structures of a complete (linear) order and on a "hub and spoke" structure respectively.

7.1. Cascading Equilibria

The set of equilibria for general power structures is rich and is not limited to the equilibrium singled out in Proposition 3. In principle, players are allowed to steal from players below them even if they are not their direct successors, but perhaps only distant ones via many intermediate agents. In principle again, this generates the possibility of complex equilibria in which some layers in the arborescence are perhaps skipped in the expropriation process while others are not. Fortunately, these features of equilibria can all be ruled out.

All the equilibria of our model are *cascading equilibria* in the sense that stealing "cascades up" in the arborescence including all the players who do not cooperate. From the point of view of a given player i, stealing players who are at the top of a "chain of stealing" in $\mathcal{B}_i$ will in general be the ones that have accumulated a larger net stolen wealth balance, and hence are more advantageous to steal from given the class of cost of stealing functions we have postulated.

Cascading equilibria are best defined by looking at a specific $t = 1$ subgame, after co-operation choices have been made at $t = 0$ and then observed by all. As we noted above, a $t = 1$ subgame is identified by a subset $\mathcal{Z} \subset \mathcal{P}$ of non-cooperating players. Without loss of generality $\mathcal{Z}$ is a strict subset of $\mathcal{P}$ since all powerless players are considered cooperating players.

Given a $\mathcal{Z}$, the original partial order $\succ$ defines one or more *sub-arborescences* on $\mathcal{Z}$, which we denote $(\mathcal{Z}, \succ)$.[24] The sub-arborescences $(\mathcal{Z}, \succ)$ each have a root,[25] moreover a player is a successor of another player in the relevant sub-arborescence if and only if it is a successor of the given player in the original partial order $\succ$. Armed with these observations we can proceed with a formal definition.

Definition 1. *Cascading Equilibria: Fix a $t = 1$ subgame, identified by the set $\mathcal{Z}$ of non-cooperating players. An equilibrium in subgame $\mathcal{Z}$ is a cascading equilibrium if any player in $\mathcal{Z}$ who has one or more immediate successors in $\mathcal{Z}$ only steals from his immediate successors in $\mathcal{Z}$.*

An equilibrium is a cascading equilibrium if it is a cascading equilibrium in every subgame.

We can now make our main claim formal.

Proposition 4. *All Equilibria Are Cascading Equilibria: All equilibria of the model are cascading equilibria in the sense of Definition 1.*

[24]See Definition A.1. Notice also that this is a slight abuse of notation since the partial order restricted to $\mathcal{Z}$ is a different object than $\succ$. This will not cause any ambiguity here.

[25]If $1 \in \mathcal{Z}$ the sub-arborescence is unique. If $1 \notin \mathcal{Z}$ there are multiple *disjoint* sub-arborescences.

Proof: See Section A.7 in the Appendix.

The following is obvious given Proposition 4, but it is worth re-empasizing at this point.

Remark 6. *Cascading Equilibria With Enforcement: The equilibria used to prove Proposition 2 constructively are Cascading Equilibria.*

The equilibrium with Centrally Enforced Cooperation of Proposition 3 is a Cascading Equilibrium.

Knowing that all equilibria are cascading equilibria allows a further characterization of the set of equilibria in our model via a key observation that is embodied in Proposition 5 below.

It is not hard to outline intuitively what the observation is. Suppose that in some cascading equilibrium we found three non cooperating players — say i_1, i_2 and i_3 — "ranked in a row" in the arborescence, with no other non cooperating player in between. Since we are in a cascading equilibrium player i_1 would be stealing only from i_2 while i_2 would be stealing only from i_3, and i_3 would be stealing from some player in $\mathcal{B}_{i_3}$ that could be cooperating if there are no non cooperating players in $\mathcal{B}_{i_3}$.

Using Proposition 1 and (11) it is then not hard to see that the amounts stolen by these three players would be as follows. Player i_3 would steal some amount $z_{i_3} \geq z(0)$ from one or more players in $\mathcal{B}_{i_3}$. Player i_2 would steal some amount $z_{i_2} > z_{i_3}$ from player i_3, while player i_1 would steal some amount $z_{i_1} > z_{i_2}$ from player i_2. However, player i_2 would then have an incentive to deviate and cooperate at $t = 0$. Indeed if he did so, in the ensuing subgame the player in $\mathcal{B}_{i_1}$ with the largest accumulated stolen wealth would no longer be i_2, but would instead be i_3. Hence after the deviation, player i_1 would "skip" player i_2 and only steal from player i_3. Therefore after the deviation player i_2 would enjoy a payoff of $\tilde{\omega}_{i_2}$. This is obviously greater than his payoff before the deviation.[26] Hence we conclude that the configuration we hypothesized is *not* possible in a cascading equilibrium. To make this key observation formal, we first proceed with a definition.

Definition 2. *Attack Triple: Consider a General Power Structure as in Assumption 1 and let Assumption 2 hold.*

Consider a subset $\mathcal{Z} \subset \mathcal{P}$ of players that contains no powerless players in $(\mathcal{P}, \succ)$.

[26] Before the deviation the payoff to i_2 is bounded below by $\omega_{i_2} + z_{i_2} - z_{i_1} < \omega_{i_2} < \tilde{\omega}_{i_2}$.

We say that $\mathcal{Z}$ contains an "attack triple" if we can find a set of three players $i_1 \succ i_2 \succ i_3$ in $\mathcal{Z}$ such that for any $j \in \mathcal{Z}$, $j \neq i_1, i_2, i_3$, we have that

$$i_1 \succ j \quad \Rightarrow \quad i_3 \succ j \tag{13}$$

In other words, given an attack triple i_1, i_2, i_3, within $\mathcal{Z}/\{i_2, i_3\}$, all successors of i_1 are also successors of i_3 and all successors of i_2 are also successors of i_3.

Once we have Definition 2, we can use it to formalize the observation we have described.

Proposition 5. *No Attack Triples in Equilibrium: Consider a General Power Structure as in Assumption 1 and let Assumption 2 hold. Consider any equilibrium and let $\mathcal{Z}$ be the set of players who choose not to cooperate. Then $\mathcal{Z}$ does not contain an attack triple.*

Proof: See Section A.8 in the Appendix.

Proposition 5 has more or less "bite" depending on the power structure at hand. This is what we investigate in the next two Subsections.

7.2. Complete Orders

There is a case in which the bite of Proposition 5 is completely straightforward. When $\succ$ is a strict *complete order* so that any two players *must* be ranked we are in the "linear order" case. Without loss of generality (up to a relabeling of players) assume that $i \succ j$ if and only if $j > i$. Player 1 is the root and the other players are ranked in the order of their "names." For any i, the set $\mathcal{B}_i$ is the set of players j such that $j > i$ and the set $\mathcal{A}_i$ is the set of players j such that $j < i$.

In a linear order it is completely obvious that *any* set of three players must have the property (13) invoked by Definition 2. Hence, the next Corollary is a simple consequence of Proposition 5.

Corollary 1. *Complete Orders and Cascading Equilibria: Consider a General Power Structure as in Assumption 1 and let Assumption 2 hold. Assume that $(\mathcal{P}, \succ)$ is such that $\succ$ is a complete order.*

Consider any Equilibrium and let $\mathcal{Z}$ be the set of players who choose not to cooperate. Then $\mathcal{Z}$ contains at most two players.

When the players are arranged in a single power "chain" the absence of attack triples has maximum impact on the set of cascading equilibria.

Our next task is to examine the "polar" case to that of a complete order.

7.3. Highly Branching Power Structures

A critical feature of arborescences with a root that defines how "far" they are from a complete order is the degree to which they fan out. The direct successors of a given player are not ranked among themselves. Loosely speaking, the more direct successors the more partial (incomplete) is the strict order. Since our model is interesting when a chain of three or more players is in place, we focus attention on structures that fan out to a high degree in which the direct successors of the branching players have themselves successors of some type.

Bearing in mind these informal observations, we proceed with a canonical case of power structure in which fanning out of a high degree occurs at the root.

Proposition 6. *Many Direct Successors And Non-Cooperative Equilibria: Consider a general power structure $(\mathcal{P}, \succ)$ in which player 1 has M direct non-powerless successors and let Assumptions 1 and 2 hold.*[27]

Then there exists an $\underline{M}$ such that whenever $M \geq \underline{M}$ the game has an equilibrium in which

(i) Player 1 does not cooperate and neither does each of his direct successors.

(ii) All other players choose to cooperate.

Proof: See Section A.9 in the Appendix.

Before delving into the intuition behind Proposition 6, we pause to make an observation.

The power structure posited in Proposition 6 belongs to the class to which Proposition 1 applies. It follows that if M is sufficiently large then Proposition 1 will guarantee that *another* equilibrium with Centrally Enforced Cooperation will exist as well. Multiple equilibria are guaranteed to exist in this case.

We now come to the intuition behind the non-cooperative equilibrium singled out in Proposition 6. Each direct successor to 1, say i, expropriates an amount $z(0)$ from a powerless player in $\mathcal{B}_i$. If there are any non powerless players in $\mathcal{B}_i$ they do not deviate from cooperating at $t = 0$ since otherwise they would be expropriated by a larger amount by player i.

[27] Notice that each of these M players must have at least one powerless successor. This implies that $N \geq 2M + 1$.

Player 1 expropriates an amount z_1/M from each of his direct successors. The total amount he steals is equally distributed among M players, and using Proposition 1 we know that it (namely z_1) does not change with M. Hence the total amount stolen from each direct successor becomes arbitrarily small as M grows large. Therefore in this equilibrium each of the direct successors of player 1 receives a payoff that is arbitrarily close to what he would get if he alone stole from one of his powerless successors. By deviating and cooperating at $t = 0$, player i would instead escape expropriation by player 1 and gain $\tilde{\omega}_i - \omega_i$. However, by Assumption 2, as M becomes large, this gain will be less than the loss from not expropriating his powerless successors. Hence the deviation to cooperation at $t = 0$ is not profitable for i.

7.4. A Hub and Spoke Structure

If each of the many direct successors of player 1 has one powerless successor, the power structure can be interpreted as a "trading post" where many trading pairs ordered by power meet. At the trading post the hub player can potentially enforce their cooperating behavior, whereas in isolation each trading pair would be unable to cooperate.

Putting together Propositions 3 and 6 we know that, provided the number of participants is large enough, the equilibrium outcome can be of one of at least two varieties. One in which the central player, in equilibrium, "disciplines" the trading activity of the numerous pairs by exacting a (small) "tax" on all participants. All participants engage in surplus-creating trade in this case. The second equilibrium outcome is in a strong sense the outcome of a significant coordination failure. All non-powerless participants who can are locked into expropriation activities that prevent them from engaging in surplus creation.

Trading posts have figured prominently in the Economics literature. A highly incomplete list begins at least with the classics Wicksell (1898) and Walras (1874), and then includes the more modern Starr (2008), Shapley and Shubik (1977) and Hahn (1973).[28]

The main purpose of trading posts in the Economics tradition is roughly speaking to overcome the lack of "double-coincidence of wants" in the sense of Jevons (1875). Of course, this cannot be explicitly modeled here since wealth in our model is a single homogenous good. The insight that we gain from the model we study here is that in order to function

[28]Scholars across a variety of fields have had an interest in trading posts. We simply hope to scratch the surface in a meaningful way mentioning Anthropologists (Brøgger, 2009, Kelley, 1985, McC. Adams, 1974), Archaeologists (Malinowski, 1974), Ethnographers (Turck and Turck, 1992) and Historians (Hämäläinen, 1998).

with appropriately enforced transactions, trading posts must reach a critical size in terms of the number of participants.

To fit the trading post structure into the model we have put forward, we posit that the hub is player 1 and there are a total of $2K+1$ players (including 1). The power structure is such that $1 \succ i$ for all $i \geq 2$, and for each $k = 1, \ldots, K$, $2k \succ 2k+1$ while each player $2k+1$ is powerless.

While player 1 is the "hub," each of $(2k, 2k+1)$ with $k = 1, \ldots, K$ is a trading pair (with one dominant partner) that is connected to 1 and to no other players directly.

The equilibrium in which all players but 1 engage in surplus-producing cooperation is a simple corollary of Proposition 3.

Corollary 2. *Hub Player As Enforcer: Consider $N = 2K+1$ players arranged on a hub and spoke structure as above. Let Assumptions 1 and 2 hold.*

Then there exists an $\underline{K}$ such that whenever $K \geq \underline{K}$ the game has an equilibrium in which player 1 does not cooperate while all other non-powerless players cooperate.

The equilibrium in which all non-powerless players are trapped in a coordination failure that prevents all surplus-producing activities is a simple corollary of Proposition 6.

Corollary 3. *Hub and Spoke Non-Cooperative Equilibrium: Consider $N = 2K+1$ players arranged on a hub and spoke structure as above. Let Assumptions 1 and 2 hold.*

Then there exists a $\underline{K}$ such that whenever $K \geq \underline{K}$ the game has an equilibrium in which Player 1 does not cooperate and each Player $2k$ for every k also chooses not to cooperate.

Whether a trading post should flourish into a hub of surplus-producing cooperation activities with a central enforcer, or flounder (and perhaps thus disappear) under the weight of a coordination failure is clearly beyond the scope of a simple stylized model like the one analyzed here.

8. Summary of Results

We study a jungle in which players are arranged in a strict partial order. Players above can expropriate those below in the partial order at a cost.

At time $t = 0$ players choose whether to cooperate and engage in surplus generating activities[29] (e.g. trade) or acquire the means to expropriate players below, and the decision is irreversible. Those who have acquired the offensive means then take expropriation decisions at $t = 1$.

The expropriation cost should be neither too large nor too small. Not too small so that a player does not want to expropriate all players below of all their wealth. Not too large relative to the gains from cooperation so that cooperation should not become the optimal choice even when all other players are cooperating — the "curse of plenty."

We posit a cost of stealing that captures the fact that stealing from a player that has accumulated more wealth by stealing from others has a lower marginal cost. Critically, this fits a set up in which the object of stealing is storable and divisible. It does not fit a world in which the object of stealing is perishable since in this case stealing from others what they have stolen from others makes little sense.

Under these restricted (but not knife edge) conditions, we find that any jungle, in which a player is more powerful than all others and which is populated by sufficiently many players has an equilibrium that we interpret as one in which cooperation is centrally enforced. This is so regardless of the details of the power structure in that the condition on the number of players does not depend on them. In the equilibrium with centrally enforced cooperation the most powerful player expropriates a smaller amount from all other players who in turn all cooperate and engage in surplus-producing activities. The temptation to deviate and steal from players below for all players aside from the enforcer is offset by the fact that if they did so then the enforcer would steal a larger amount from them.

Our power structures all have a unique most powerful player. It is worth emphasizing here that this is largely for simplicity. If the power structure has a top echelon that takes the place of our top player then under suitable conditions centrally enforced equilibria would emerge. Clearly these equilibria would require essential coordination among the players in the top echelon.

Finally, we are able to characterize further the set of possible equilibria. The key feature of the power structure that drives the possibility of cooperative and non-cooperative outcomes in equilibrium is how highly fanning out it is. In the case of a non-branching structure (a linear, complete order) the number of players who do not cooperate in equilibrium is at most

[29] See footnote 14 above.

two. In the polar case in which one player has many direct successors in the power structure the number of players who do not cooperate in equilibrium can grow without bound.

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Appendix

A.1. Arborescences

Definition A.1. *Arborescences and Sub-Arborescences: Consider an arborescence $(\mathcal{P}, \succ)$ with root 1. Let $i \neq 1$ be a player such that $\mathcal{B}_i \neq \emptyset$. Let $\mathcal{P}_i = i \cup \mathcal{B}_i$. Then $(\mathcal{P}_i, \succ)$, where $\succ$ is restricted to $\mathcal{P}_i$, is a* sub-arborescence with root i.

Notice that, given an arborescence $(\mathcal{P}, \succ)$ and a proper subset $\mathcal{Z}$ of $\mathcal{P}$, the set $\mathcal{Z}$ together with the restriction of $\succ$ to $\mathcal{Z}$ is either a sub-arborescence with a root, or is a collection of disjoint sub-arborescences each with a root.

We use the term sub-arborescence to indicate those obtained from $(\mathcal{P}, \succ)$ restricting attention to a $\mathcal{Z} \subset \mathcal{P}$. With a slight abuse of notation we indicate by $(\mathcal{Z}, \succ)$ the collection of sub-aborescences obtained by restricting attention to $\mathcal{Z}$.

If $i \succ j$, $i, j \in \mathcal{P}$ and, for no $h \in \mathcal{P}$, $i \succ h \succ j$, we say that i is an immediate predecessor of j and j is an immediate successor of i.

Definition A.2. *Paths and Lengths: A path in $(\mathcal{P}, \succ)$ is a sequence $(i_1, ..., i_L)$ where i_{l-1} is an immediate predecessor of i_l, $l = 2, ..., L$. The length of the path is L.*

When a path is entirely contained in a sub-arborescence we say that it is a path in that sub-arborescence.

A.2. Sufficiently Large Endowments and Interiority

Definition A.3. *Undominated Choices: Fix a $\mathcal{C}(\cdot)$ and hence a $\mathcal{C}^1(\cdot)$ and a $\mathcal{C}^2(\cdot)$. We say that s is an* undominated *expropriation choice iff*

$$s \leq \overline{z} = \operatorname*{argmax}_x \{x - \mathcal{C}^1(x)\} \tag{A.1}$$

The reason to define $\overline{z}$ as in (A.1) and to term any expropriation choice not exceeding $\overline{z}$ an undominated choice is that from (11) and the fact that by assumption $\mathcal{C}^2(\cdot)$ is non-negative we must have that

$$\overline{z} \geq z(h) \qquad \forall\, h \tag{A.2}$$

Hence no player will optimally choose to expropriate an amount exceeding $\overline{z}$, regardless of the choice of all other players.

Next, we define what is meant by endowments that are "sufficiently large." It should be noted that the lower bound we state is by no means tight for our purposes. We proceed in this way for the sake of transparency and simplicity.

Definition A.4. *Large Endowments: We say that the players's endowments are* sufficiently large *if*

$$\omega_i \quad \geq \quad (N-1)\,\overline{z} \qquad\qquad \forall\, i \geq 2 \tag{A.3}$$

Proposition A.1. *Interiority: Assume that endowments are sufficiently large according to Definition A.4. Consider a non powerless player i and any expropriation choices s_{-i} by all players other than i that are undominated according to Definition A.3. Next, use (4) to compute h_j for every $j \in \mathcal{B}_i$.*

Then the problem of player i as stated in (9) with the added constraints

$$\omega_j \;+\; h_j \;-\; \sum_{\substack{q \in \mathcal{A}_j \\ q \neq i}} s_{qj} \;-\; s_{ij} \;\geq\; 0 \qquad \forall\, j \,\in\, \mathcal{B}_i \tag{A.4}$$

has the same solution as the original one since none of the constraints in (A.4) are in fact binding.

Proof: By definition $h_j \geq 0$. Since the choices in s_{-i} are undominated we know that $s_{qj} \leq \overline{z}$ for every $q \in A_j$ with $q \neq i$. Since endowments are sufficiently large we know that $\omega_j \geq (N-1)\,\overline{z}$. Hence (A.4) must be satisfied for any s_{ij} that is itself undominated and hence does not exceed $\overline{z}$. ∎

A.3. The Proof of Proposition 1

Proof of Proposition 1: Consider Problem (9). This clearly may have multiple solutions. Let a typical solution be denoted by $s_i^* = \{s_{ij}^*\}_{j \in \mathcal{B}_i}$. By inspection, it is clear that

$$j \notin \mathcal{M}_i \quad \Rightarrow \quad s_{ij}^* \;=\; 0 \tag{A.5}$$

By strict concavity $\sum_{j \in \mathcal{B}_i} s_{ij}^*$ is uniquely determined. Using (11) we obviously have that

$$z(\overline{h}_i) = \sum_{j \in \mathcal{M}_i} s_{ij}^* \;=\; \sum_{j \in \mathcal{B}_i} s_{ij}^* \tag{A.6}$$

and this suffices to prove the claim. ∎

A.4. Equilibria In Subgames

The following two definitions allow us to recursively define expropriations along the longest paths for any arbitrary subset of players $\mathcal{Z}$. We begin with an arbitrarily chosen sub-arborescence.

Definition A.5. *Longest Path Expropriations in $\mathcal{Z}$: Let $\mathcal{Z}$ be a subset of players not containing any powerless players in $(\mathcal{P}, \succ)$ and consider any one of the disjoint sub-arborescences with a root in $(\mathcal{Z}, \succ)$. Let this arbitrarily chosen sub-arborescence be denoted by $(\mathcal{K}, \succ)$ with $\mathcal{K} \subseteq \mathcal{Z}$, and let k be its root. Let L be the length of the longest path Y^k in $(\mathcal{K}, \succ)$ (which obviously originates with k), and choose one such longest path arbitrarily if necessary. Let $y^k(n)$ be the n^{th} player on this path. Obviously, $y^k(L)$ is the last player on such path and has no successors in $(\mathcal{K}, \succ)$, but since powerless players are excluded from $\mathcal{Z}$, it must have one or more successors in $(\mathcal{P}, \succ)$.*

Given such path Y^k we define recursively expropriation decisions s_{ij}^* for players along on the path. Let player $y^k(L+1)$ be an arbitrary successor of player $y^k(L)$. Next, define

$$s^*_{y^k(L), y^k(L+1)} \;=\; z(0) \tag{A.7}$$

and then recursively backwards for $n = 1, \dots, L-1$

$$s^*_{y^k(n),y^k(n+1)} \ = \ z(s^*_{y^k(n+1),y^k(n+2)}) \tag{A.8}$$

The procedure we have just defined can be repeated recursively to cover the entire set $\mathcal{Z}$.

Definition A.6. *Recursive Expropriation Attribution for $\mathcal{Z}$: The procedure explicited in Definition A.5 is recursively repeated as follows.*

Begin by replacing $\mathcal{Z}$ with $\mathcal{Z}/Y^k$ and then selecting a new arbitrary sub-arborescence with a root in $\mathcal{Z}/Y^k$ and a longest path within it. For this path we define recursively expropriation decisions s^*_{ij} for players on the path as above. Repeating this step, we will eventually exhaust all elements of $\mathcal{Z}$. We call the array so constructed, in which $s^*_{ij} = 0$ for $i \notin \mathcal{Z}$, a recursive expropriation attribution for $\mathcal{Z}$. *We denote this by $s^*(\mathcal{Z})$ and the associated longest paths used in its construction by $\{Y^k\}$, where each player k denotes the root of each path Y^k.*

In a recursive expropriation attribution a player expropriates resources from at most one of his successors.

Remark A.1. *Increasing Recursive Stolen Amounts: Let $\mathcal{Z}$ be a subset of players not containing any powerless players. Consider a recursive expropriation attribution for $\mathcal{Z}$ as in Definition A.6. Then, using Assumption 2, simple algebra and (A.7) and (A.8) suffices to show that along any of the longest paths Y^k used in its construction we must have that for $n = 1, \dots, L-1$.*

$$s^*_{y^k(1),y^k(2)} \ > \ \dots \ > \ s^*_{y^k(n),y^k(n+1)} \ > \ \dots > s^*_{y^k(L),y^k(L+1)} > \ \tilde{\omega}_i \ - \ \omega_i \tag{A.9}$$

for every i. Furthermore when endowments are sufficiently large according to Definition A.4 (see Proposition A.1)

$$s^*_{y^k(n),y^k(n+1)} < s^*_{y^k(n+1),y^k(n+2)} + \omega_i$$

and

$$s^*_{y^k(L),y^k(L+1)} < \tilde{\omega}_i.$$

We now proceed to prove a preliminary result on the paths associated with recursive expropriation attributions.

Lemma A.1*: Let $\mathcal{Z}$ be a subset of players not containing any powerless players. Consider a recursive expropriation attribution $s^*(\mathcal{Z})$ for $\mathcal{Z}$ and the associated collection of longest paths used in its construction, with typical element Y^k. If $i \in Y^k$ and $j \in Y^{k'}$, where $k \neq k'$ and j is a successor of i, then every player in $Y^{k'}$ is a successor of i. Furthermore, the length of $Y^{k'}$ does not exceed the number of successors of player i in Y^k.*

Proof: First notice that by the definition of sub-arborescence, either $i \succ k'$ or $k' \succ i$. If the former, the claim is proved. If the latter, then either $k \succ k'$ or $k' \succ k$. If $k \succ k'$, Y^k was constructed before $Y^{k'}$ and adding k' would have yielded a longer path. Conversely, if $k' \succ k$ then $Y^{k'}$ was constructed before Y^k and adding k would have yielded a longer path. Hence, we have shown that k' is a successor of i. Therefore, Y^k was constructed before $Y^{k'}$. The latter part of the claim follows noting that if the length of $Y^{k'}$ exceeds the number of successors of player i in Y^k, then Y^k is not a longest path. ∎

We now state and prove a central lemma. Let $\mathcal{Z}$ be a subset of players not containing any powerless players. The list of players in $\mathcal{Z}$ identifies a subgame in which all players in $\mathcal{Z}$ do not cooperate. Let this subgame be denoted by $\boldsymbol{\tau^1}(\mathcal{Z})$.

Lemma A.2: *Let $\mathcal{Z}$ be a subset of players not containing any powerless players. Any recursive expropriation attribution $s^*(\mathcal{Z})$ for $\mathcal{Z}$ is a Nash equilibrium of the subgame $\boldsymbol{\tau^1}(\mathcal{Z})$.*

Proof: The proof of the claim consists in checking that the behavior in the statement of the Lemma constitutes a best response to the behavior of others for all players in $\mathcal{Z}$, with the best responses as characterized in Proposition 1.

Let an $s^*(\mathcal{Z})$ as in the statement of the Lemma be given and consider the associated collection of longest paths used in its construction, with typical element Y^k. Select a player $i \in \mathcal{Z}$ and $Y^k = \{y^k(n)\}_{n=1}^L$ such that $y^k(\hat{n}) = i$. Then, by our contruction and Lemma A.1 when $\hat{n} = L$, $j \notin \mathcal{Z}$ for any successor j of player i. Thus, $s^*_{y^k(L),y^k(L+1)} = z(0)$ for one arbitrary successor $y^k(L+1)$ is a best reply by Lemma 1 since $\underline{h}_{y^k(L)} = 0$. If $\hat{n} < L$, again by our construction and Lemma A.1 setting

$$s^*_{y^k(\hat{n}),y^k(\hat{n}+1)} \;=\; z\big(s^*_{y^k(\hat{n}+1),y^k(\hat{n}+2)}\big)$$

and $s^*_{y^k(\hat{n}),j} = 0$ for any $j \neq y^k(\hat{n}+1)$ is a best reply by Lemma 1 since $\underline{h}_{y^k(\hat{n})} = -s^*_{y^k(\hat{n}+1),y^k(\hat{n}+2)}$. ∎

Finally, we notice that the equilibria identified in Lemma A.2 may not be the only equilibria in a given subgame.

Remark A.2. Multiple Equilibria in Subgames: *In constructing the recursive expropriation arrays we arbitrarily assumed that a player expropriates only one of his successors even when multiple longest paths may be available. He may obtain the same payoff expropriating several of his successors. The total amount expropriated will not vary, but the amounts seized from each successor may. Proposition 1 suffices to pin down the aggregate amount, which can then be apportioned arbitrarily across the successors q of player i for whom h_{iq} is minimal as in (10).*

A.5. The Proof of Proposition 2

The proof of Proposition 2 is constructive. In particular, we will show that a subgame perfect equilibrium exists in which a (Cascading) pure strategy Equilibrium, as in Definition 1 (Subsection 7.1) is played in every subgame.

Proof of Proposition 2: To begin with, let

$$\hat{\mathcal{C}} = \mathcal{C}^1[z(0)] + z(0)\,\mathcal{C}^2(0) \tag{A.10}$$

Next, we distinguish two cases.

Case 1*:* Suppose that for some non-powerless $i^* \neq 1$ we have that

$$\omega_{i^*} + z(0) - z[\,z(0)\,] - \hat{\mathcal{C}} \geq \tilde{\omega}_{i^*} - z(0) \tag{A.11}$$

We then construct a SPE as follows. At time $t = 1$, players 1 and i^* choose not to cooperate while all other players choose to cooperate.

In the subgame following these $t = 1$ choices, the following equilibrium is played. Player i^* expropriates the amount $z(0)$ from a powerless successor, while player 1 expropriates the amount $z[z(0)]$ from player i^*. Whether player i^* expropriates one or several powerless successors is inessential as long as the total amount that is seized is $z(0)$.

Let's now consider subgames that follow deviations. We will specify the corresponding expropriation arrays and verify that unilateral deviations do not increase payoffs.

(i) If player i^* deviates unilaterally and chooses to cooperate then player 1 seizes $z(0)$ from player i^*. Therefore in this case player i^* obtains a payoff equal to the right-hand side of (A.11). Since the payoff if he does not deviate is equal to the left-hand of the same inequality, the deviation is not profitable.

(ii) Using Assumption 2, it is immediate to check that player 1 does not want to deviate unilaterally and cooperate.

(iii) If a successor i of player i^* who is not powerless chooses to not cooperate a cascading equilibrium, as in Definition 1, is played. The payoff of player i is equal to $\omega_i + z(0) - z[\,z(0)\,] - \hat{\mathcal{C}}$. Since $z(0) - z[\,z(0)\,] < 0$ and $\omega_i < \tilde{\omega}_i$ such deviation is unprofitable since the payoff of player i when cooperating is $\tilde{\omega}_i$.

(iv) If a player i that is not a successor of player i^* chooses to activate his power an equilibrium is played in which player 1 expropriates $z[\,z(0)\,]$ from player i and nothing from player i^*. Players i and i^* each expropriate $z(0)$ from one of their successors, which are of course distinct. After the deviation, the payoff of player i is equal to $\omega_i + z(0) - z[\,z(0)\,] - \hat{\mathcal{C}}$. As in (iii) this is smaller than his payoff before the deviation, namely $\tilde{\omega}_i$.

(v) In any other subgame a Cascading Equilibrium, as in Definition 1, of that subgame is played. Any such subgame entails deviations that are not unilateral and hence no further considerations about payoffs are needed.

Hence, a pure strategy subgame perfect equilibrium exists in this case. Notice that a Cascading Equilibrium is played in all subgames, on or off the equilibrium path.

Case 2*:* Suppose that (A.11) is false in the sense that for every $i \neq 1$ we have that

$$\omega_i + z(0) - z[\,z(0)\,] - \hat{\mathcal{C}} < \tilde{\omega}_i - z(0) \tag{A.12}$$

It is then obvious that in this case we have

$$\omega_i + z(0) - z[z(0)] - \hat{\mathcal{C}} < \tilde{\omega}_i - \frac{z(0)}{N-1} \qquad \forall\, i \neq 1 \tag{A.13}$$

In this case, we construct an SPE as follows. At time $t = 1$ player 1 decides not to cooperate and every other player chooses to cooperate. Player 1 steals an amount

$$\frac{z(0)}{N-1} \tag{A.14}$$

from each of his $N - 1$ successors. Notice that in the subgame that ensues, no player other than player 1 chooses a positive expropriation amount and that by Proposition 1, the choice of player 1 is optimal. Finally notice that player 1 does not wish to deviate unilaterally by Assumption 2.

In any other subgame following a unilateral deviation by a player $i \neq 1$, player 1 then expropriates $z[z(0)]$ from player i, and player i expropriates $z(0)$ from one of his successors. The payoff of player i is equal to the left-hand side of (A.13) and is smaller than the payoff from cooperating which is of course is equal to the right hand side of (A.13).

Since, Case 1 and Case 2 are exhaustive of all possibilities, the proof is now complete. ∎

A.6. The Proof of Proposition 3

Proof of Proposition 3: The claim follows from case (2) in the proof of Proposition 2 noting that since $\omega_i < \tilde{\omega}_i$ and $z(0) - z[z(0)] < 0$ the inequality (A.13) holds when N is sufficiently large. ∎

A.7. The Proof of Proposition 4

Proof of Proposition 4: Fix a $t = 1$ subgame, identified by the set $\mathcal{Z}$ of non-cooperating players. We need to show that any equilibrium in subgame $\mathcal{Z}$ is a cascading equilibrium.

Consider any player $k \in \mathcal{Z}$ and the arborescence $(\{k\} \cup (\mathcal{B}_k \cap \mathcal{Z}), \succ)$ having root k.

It suffices to show that in any such arborescence if $\mathcal{B}_k \cap \mathcal{Z}$ is not empty, player k steals only from players[30] such as a j for whom there is a longest path Y^k in $\{k\} \cup (\mathcal{B}_k \cap \mathcal{Z})$ such that $y^k(2) = j$, and if $\mathcal{B}_i \cap \mathcal{Z}$ is empty (which of course must be true for at least one player in $\mathcal{B}_k$) player k steals from one or more players in $\mathcal{B}_k/\mathcal{Z}$.

The latter is trivial, so suppose that $\mathcal{B}_k \cap \mathcal{Z}$ is not empty and suppose that the claim is true for all the successors of player $k \in (\{k\} \cup (\mathcal{B}_k \cap \mathcal{Z}), \succ)$.

Then, by repeating the arguments in Remark A.1, the amount stolen by such player $j = y^k(2)$ is strictly larger than the amount stolen by any player in $\mathcal{B}_k$ for whom such longest path does not exist. Hence, it is optimal for k to steal only from players such as j. This shows that in any equilibrium of the subgame this must in fact be the case and therefore it suffices to prove the claim. ∎

[30] See Remark A.2.

A.8. The Proof of Proposition 5

Proof of Proposition 5: The argument is by contradiction. Suppose that an equilibrium with an attack triple exists. Let $\mathcal{Z} \subseteq \mathcal{P}$ be the set of players who choose not to cooperate on the equilibrium path. Let (i_1, i_2, i_3) be the attack triple within $\mathcal{Z}$ that is assumed to exist. We will show that i_2 has an incentive to deviate unilaterally and choose to cooperate.

We begin by noticing that (13) on any longest path within $\mathcal{Z}$ starting with i_1 has i_2 next and then i_3, and the longest path within $\mathcal{Z}$ starting with i_2 has i_3 next.

By Remark A.1 the equilibrium expropriation array s^* is such that

$$s^*_{i_1,i_2} > s^*_{i_2,i_3}$$

We can then conclude that the payoff to i_2 in this equilibrium is smaller than $\tilde{\omega}_{i_2}$. If i_2 were instead to uninaterally deviate and cooperate, in the subgame that follows this deviation any longest path within $\mathcal{Z}/i_2$ starting with i_1 has i_3 next. Since after this deviation i_2 earns $\tilde{\omega}_{i_2}$, the deviation is profitable and the proof is now complete. ∎

A.9. The Proof of Proposition 6

Proof of Proposition 6: Without loss of generality suppose that the M direct successors of 1 are players $2, \ldots, M+1$.

We construct a SPE as follows. At time $t = 1$, players $1, ..., M+1$ choose not to cooperate while all other players choose to cooperate.

In any subgame where $\mathcal{Z} \neq \{1, ..., M+1\}$ a Cascading Equilibrium is played.

In the subgame where $\mathcal{Z} = \{1, ..., M+1\}$, each player in $\mathcal{Z}$ different from player 1 expropriates $z(0)$ from his powerless successors, and Player 1 steals an amount $z[\,z(0)\,]\,/\,M$ from each of his immediate successors.

It is easy to check that by Assumption 2 player 1 does not want to deviate and receive $\tilde{\omega}_1$ by cooperating. Also by Assumption 2, the same holds for sufficiently large M for players $i = 2, ..., M$ since their payoff is

$$\omega_i + z(0) - \frac{z[\,z(0)\,]}{M} - \mathcal{C}^1(z(0)) - z(0)\,\mathcal{C}^2(0)$$

and the amount $z[\,z(0)\,]/M$ can be made arbitrarily small.

Consider now any non-powerless player i who is not an immediate successor to 1. Notice that there may or may not be any, and if there are none there is nothing more to prove. Let m_i be the player that is an immediate successor of 1 and also a predecessor of i. In the subgame with $\mathcal{Z} = \{1, ..., M+1\}$ the payoff to i is simply $\tilde{\omega}_i$ as m_i expropriates from powerless successors. If i deviates unilaterally, in the subgame with $\mathcal{Z} = \{1, ..., M+1, i\}$, in the cascading equilibrium that is played m_i will steal $z[\,z(0)\,]$ from i, and i will steal $z(0)$ from his successors. Hence, by deviating player i gets

$$\omega_i + z(0) - z[\,z(0)\,] - \hat{\mathcal{C}} \tag{A.15}$$

whereas if he does not deviate he gets $\tilde{\omega}_i$. Since $\tilde{\omega}_i > \omega_i$ we conclude that this unilateral deviation cannot be profitable for i. ∎

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