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## **Dynamic reform of public institutions: A model of motivated agents and collective reputation**

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Abstract

***Dynamic reform of public institutions: A model of motivated agents and collective reputation\****

State capacity is optimized when public institutions are staffed by individuals with public-service motivation. However, when motivated agents value the collective reputation of their place of employment, steady-state equilibria with both high and low aggregate motivation (reputation) in the mission-oriented sector exist. Reforming a low-motivation institution requires a non-monotonic wage path: since the effect of higher wages on motivation is negative for a high-reputation institution, but positive for a low-reputation institution, a transition to a high-reputation steady state requires an initial wage increase to crowd motivated workers in, followed by a wage decrease to crowd non-motivated workers out.

*Keywords:* Motivated Agents, Institutional Reform, Public Sector.

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# 1 Introduction

There are three kinds of [nonmaterial] rewards: a sense of duty and purpose, the status that derives from individual recognition and personal power, and the associational benefits that come from being part of an organization (or a small group within that organization) that is highly regarded by its members or by society at large. ~ James Q. Wilson, *Bureaucracy* (1989)

While most economists would agree that state capacity is an important factor for economic performance, there is little consensus on how state capacity can be developed. Arguably, a key element of state capacity is a well-functioning public sector that efficiently allocates resources to public goods and services, and limits distortionary factors such as corruption and elite capture that divert public funds away from socially beneficial outlets. In turn, efficiency within the public sector is optimized when public institutions are staffed by individuals with a motivation for public service, who share the mission (objective) of the institution. This suggests that policies aimed at selecting motivated workers into the public sector offer a promising way to reform underperforming public institutions.

The issue of selection into the public sector, or other mission-oriented institutions, has received significant attention in recent economic literature: The theoretical literature on public-sector motivation has highlighted the prediction that motivated individuals will disproportionately select into mission-oriented institutions (Francois, 2000 and Besley and Ghatak, 2005, see Francois and Vlassopoulos, 2008 for an overview), and empirical studies in Western Europe have confirmed that public-sector employees exhibit higher relative levels of motivation and altruism than private-sector employees (Gregg et al., 2011 and Dur and Zoutenbier, 2014). However, recent research has provided evidence of reverse sorting in some developing countries with public sectors perceived to be of poor quality: Hanna and Wang (2013) and Banerjee et al. (2015) find that prospective public-sector employees in India are more likely to engage in corrupt behavior than their peers. Moreover, Cowley and Smith (2013) document that, after controlling for demographic characteristics, public-sector workers report lower levels of motivation than private-sector workers in twenty-one countries, and these countries feature relatively high levels of corruption on average (see Finan et al., 2015 for an overview of the empirical literature on selection into the public sector).

Given the predictions of the theoretical literature, these new empirical results raise the question of why some public institutions appear to be in equilibria where motivated individuals disproportionately select *out* of employment in the mission-oriented sector. Additionally, given the role selection plays in providing state capacity, an equally important question is how under-performing institutions can be reformed

to bring about a transition to an equilibrium with efficient sorting.

In this paper, we show that the puzzle of multiple equilibria can be resolved by considering a model of labor-market sorting with motivated agents who value both mission and collective reputation. Additionally, we analyze a dynamic version of the model and highlight the novel prediction that the effect of wages on motivation is conditional on the reputation of the mission-oriented institution, which implies that a non-monotonic wage path is needed to transition from a low-motivation to a high-motivation steady-state.

The assumption that motivated employees value the collective reputation of their employer is supported by the literature on identity and social image: As argued by Akerlof and Kranton (2005), employees may directly value the identity associated with their job, and will logically seek employment in institutions consistent with their personal identity. Additionally, the collective reputation of an institution can affect worker choice through the channel of prosocial signaling (à la Bénabou and Tirole, 2006, and Ariely et al., 2009) since the collective reputation, or aggregate behavior, of an institution provides a signal of its employees' type.<sup>1</sup> Lastly, referring explicitly to bureaucracies, the above quote from James Q. Wilson suggests that both mission (purpose) and reputation matter to public-sector employees: while a motivated worker might find a job in a well-regarded non-governmental organization attractive, they might be negatively disposed towards working for a police force widely viewed as corrupt.

To summarize, the model relies on two key assumptions: (i) there exists a motivated type who, all else equal, has a higher productivity in mission-oriented institutions; and (ii) the motivated type values the collective reputation of the institution, deriving positive value from a high reputation and negative value from a low reputation.<sup>2</sup> While we remain agnostic as to the precise mechanism behind this behavioral element, the model we construct is consistent with a micro-foundation based on identity payoffs, prosocial signaling, or homophily.<sup>3</sup>

We first show that the model implies multiple equilibria – both low-motivation

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<sup>1</sup>A related argument derives from social interaction in the workplace: given a correlation between the collective reputation and workforce composition of an institution, value homophily in the workplace (à la Lazarsfeld and Merton, 1954) provides yet another explanation why workers may value collective reputation. Also, see Henderson and Steen (2015) for an in-depth discussion regarding the role of a firm's reputation in complementing the prosocial-identity of its employees.

<sup>2</sup>Following Tirole (1996), we define collective reputation as the average behavior within the institution. However, in our model, aggregate behavior is a linear function of the proportion of motivated individuals, which allows us to define collective reputation as either aggregate behavior or aggregate work-force composition.

<sup>3</sup>Non-motivated types may also value the collective reputation of the mission-oriented institution because of reputation concerns à la Bénabou and Tirole (2011); our analysis assumes that the motivated type places a greater weight on the collective reputation than the non-motivated type or, equivalently, that both types are characterized by some degree of homophily.

equilibria and high-motivation (efficient) equilibria may exist for given parameter values. This multiplicity is intuitive, given that motivated types effectively value an assortative labor-market match. Generally, a high-motivation equilibrium is characterized by a higher proportion of motivated types and a *lower* institutional wage than in the low-motivation equilibrium. The reason a lower wage is maintained in the high-motivation equilibrium is that motivated types are compensated by the high-motivation collective reputation (in addition to any mission payoffs), while the low wage deters non-motivated types from entering the mission-oriented institution.<sup>4</sup>

Several articles have highlighted examples of multiple equilibria in models with motivated agents (see for example Caselli and Morelli, 2004, Macchiavello, 2008, Kosfeld and von Siemens, 2011, and Aldashev et al., 2014). The most novel contribution of our paper is that we formally analyze the problem of reforming an institutional culture. Specifically, we ask the question of how a mission-oriented institution with low reputation and a relatively low proportion of motivated workers can transition to an efficient, high-motivation steady-state equilibrium. The policy tool we consider for enacting a transition is the relative wage; wages can be changed transparently and are a commonly utilized policy tool for instituting public-sector reform (both in theory, e.g. Besley and McLaren, 1993, and in practice, see Rose-Ackerman, 1999 pp. 71-75).

To analyze this question formally, we consider a dynamic version of the model and demonstrate the novel prediction that the effect of a wage change on motivation *depends on the initial reputation* of the mission oriented institution: starting from a reputation of low-motivation, an increase in the wage *increases* aggregate motivation, while starting from a reputation of high-motivation, an increase in the wage *decreases* motivation. The intuition behind this result lies in the fact that, holding aggregate motivation constant, a wage increase causes an equal proportion of non-motivated and motivated workers to enter the mission-oriented institution. Therefore, if the mission-oriented institution begins with a low average level of motivation, then the proportion of motivated workers increases with a wage increase – leading to an increase in average motivation and making the mission-oriented institution even more attractive to motivated workers. By the same mechanism, however, if the mission-oriented institution begins with a high average level of motivation, than a higher wage decreases average motivation.

This result also provides an explanation for the contrasting empirical results regarding the effect of wages on motivation. Two recent randomized control trials (RCTs) vary wages for mission-oriented institutions and measure the resulting effect on the motivation of applicants: Dal Bó et al. (2013) randomized wages for a

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<sup>4</sup>Analogous to the efficiency wages in Handy and Katz, 1998, Besley and Ghatak, 2005, and Delfgaauw and Dur, 2007.

position as a community development agent for a program in Mexico and find that higher wages have a *positive* impact on public sector motivation. However, Deserranno (2015) randomized expected earnings for a position as a community health promoter for a program in Uganda and finds that higher wages have a *negative* impact on prosocial motivation.

Notably, a key difference between the two RCT's is the nature of the employers: while the program in Mexico was directly administered by the Mexican government, ranked 103rd out of 175 countries in the Corruption Perceptions Index (Transparency International, 2014), the program in Uganda was administered by BRAC, which is second on the *Global Journal's* rankings of the top 500 NGOs (2015). Therefore, assuming that the Mexican public sector has a low reputation while BRAC has a high reputation, the finding that higher wages crowd out motivation in Uganda, but crowd in motivation in Mexico are consistent with the theoretical finding that the effect of wages on motivation depends on the underlying reputation of the mission-oriented institution.

The conditional effect of wages on motivation also imply that, in contrast to the findings of previous papers, a high-motivation steady state equilibrium cannot be achieved by simply setting a low wage. The low wage is a feature, rather than a cause, of the high-motivation equilibrium. Instead, we show that a transitional wage path generally involves an initial increase in the wage to attract more motivated individuals into the mission-oriented institution (crowding in motivation), followed by a gradual decrease of the wage to make the institution less attractive to non-motivated individuals (crowding out non-motivation).<sup>5</sup>

The intuition for a non-monotonic transitional wage path follows from the relationship between motivation, wages and reputation outlined above: Starting from a point of low motivation, only an increase in the wage will increase average motivation. Wage increases alone, however, cannot cause a transition to the efficient, low-wage equilibrium – after the level of motivation reaches a threshold level, above which the relationship between motivation and wages switches, the wage must be gradually decreased to effect a transition to the high-motivation, low-wage equilibrium.

We emphasize that the framework we analyze is not peculiar to the public sector and NGOs: to the extent that motivated workers value the collective reputation of generic institutions, the model pertains to any firm or institution that would find it beneficial to attract motivated workers. For example, firms may seek to replicate the recruiting advantages of, say, Google, whose reputation as a dynamic and attractive employer stems at least in part from the high quality of its existing workforce; similarly,

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<sup>5</sup>While decreasing the wages of tenured workers in the public sector may be politically infeasible, an equivalent transition can be achieved by only lowering the wages offered to incoming cohorts of workers.

economics departments may seek to recruit PhD students who are motivated to join academia rather than the private sector, and these academically-motivated students may in turn value a reputation for academic placements.

Crucially, however, we show that a transition from a low reputation to a high reputation is only generally feasible if motivated workers value the mission of the relevant institution. That is, a tipping-point reputation can only be reached through a wage increase if, given a neutral reputation, motivated workers prefer employment in the institution in question over their outside option, as is the case when motivated workers directly value the social output of a public institution (i.e. mission-contingent payoffs à la Besley and Ghatak, 2005). This finding suggests that transitions are not feasible in generic institutions, and may require that a firm actively invest in, say, corporate social responsibility,<sup>6</sup> or that transitions are only possible for departments at universities with an overall reputation for academic excellence.

We also consider several relevant extensions to the baseline model. First, we show how access to commitment may enable even a generic firm to transition to a high-motivation equilibrium, as long as workers are also optimistic about the future reputation of the institution. We then consider the case of correlation between a worker's ability and their level of motivation, and show how this correlation can be leveraged to achieve a transition. This case also provides insight into a commonly-attempted strategy of creating "elite" divisions within an existing institution – for this strategy to be successful, the institution must both recruit disproportionately from an ability type with a high average level of motivation *and* offer a relatively high wage. Without a high initial wage, the strategy of recruiting from a high-motivation ability type may not be sufficient, since the overall low reputation of the institution creates an adverse selection problem.

## 1.1 Literature

In the classic study cited at the beginning of this paper, Wilson remarks that, given the lack of incentives...“what is surprising is that bureaucrats work at all” (1989). More generally, it has been argued that non-monetary incentives in the workplace play an important role in determining worker's behavior (Dewatripont et al., 1999, Akerlof and Kranton, 2000, Akerlof and Kranton, 2005, Prendergast, 2008, Huck et al., 2012, and Fischer and Huddart, 2008). A subset of this literature considers motivation in the workplace, and has largely focused on optimal contracting in the presence of a motivated type, given that non-monetary incentives can be crowded out or distorted

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<sup>6</sup>The management literature suggests that corporations engage in charitable activities for precisely this purpose; see for example Bhattacharya et al. (2008) “Using Corporate Social Responsibility to Win the War for Talent.”

by traditional monetary incentive contracts (e.g. Murdock, 2002, Dixit, 2002, Sliwka (2007) and Ellingsen and Johannesson, 2008; see Francois and Vlassopoulos, 2008 and Prendergast, 2008 for an overview).

Another strand of this literature, in which our paper arguably falls, is concerned with the question of optimal contracting with endogenous worker sorting into the mission-oriented sector, given the presence of different behavioral types. Several papers highlight that the efficiency wage in the mission-oriented sector should be low relative to the private sector, as a low wage will disproportionately attract workers with public sector motivation who are compensated by non-pecuniary benefits of public-sector employment (Handy and Katz, 1998, Francois, 2000, and Besley and Ghatak, 2005). This result has been extended to account for the fact that other facets of the public sector may disproportionately attract individuals with harmful qualities, such as laziness or antisocial motives (Delfgaauw and Dur, 2008, Auriol and Brilon, 2014), or into positions where altruism is counter-productive (Prendergast, 2007). In an article contemporary to this paper, Henderson and Steen (2015) consider a model of motivated agents who signal their prosocial identity through the reputation of their employer, where reputation is a function of the actions of the firm, and show how this can cause firms to benefit from endogenously choosing a prosocial purpose (mission).

Our main contribution to this literature is that by considering motivated agents who value the collective reputation of an institution, the question of optimal contracting is transformed from a problem of static equilibrium selection to a problem of dynamic transition, since collective reputation functions as a state variable. That is, similar to Tirole (1996), the institution and its current workers are burdened with the legacy of past behavior, which implies that the impact of incentives becomes sensitive to the institution's starting point: higher wages increase motivation in a low-motivation institution, but decrease motivation in a high-motivation institution. Therefore, reforming, say, a culture of corruption requires a more complex approach than simply replicating the incentives of a low-corruption institution.

Our paper is also related to a set of papers that analyze transitions between norms (e.g. Bisin and Verdier, 2001, Besley et al., 2014, Bidner and Francois, 2013, and Acemoglu and Jackson, 2015). For example, in a paper related to ours in spirit, Acemoglu and Jackson (2014) detail how endogenously enforced laws can be changed dynamically to transition between a steady-state of lawlessness to a steady-state of law-abiding. They show that a sudden shift in laws away from the current norm of behavior can be counter-productive, but that a series of incremental shifts can result in a transition. Our paper also shows that the path of a reform matters – a transition cannot be enacted by skipping straight to the wage of the efficient equilibrium. However, in contrast to the findings of Acemoglu and Jackson (2014), we find that

the policy tool we consider (wages) must take a non-monotonic path for the system to transition to the optimal steady state.

Lastly, we argue that our results help reconcile the well-known policy prescription of a high public-sector “efficiency wage” to deter corruption (Besley and McLaren (1993)) with the concern that higher wages will crowd out intrinsic motivation (Besley and Ghatak, 2005). Indeed both results have empirical support: higher public-sector wages are weakly correlated with lower corruption (Treisman, 2000, Van Rijckeghem and Weder, 2001, and Di Tella and Schargrodsky, 2003), and there is evidence for a below-market “public-sector efficiency wage” (see Gregg et al., 2011) in developed nations. The theoretical results of this paper help reconcile these empirical findings: In contexts where public institutions have a good reputation, higher wages will simply crowd out motivated workers. However, in contexts where public institutions have a poor reputation, say, due to high levels of corruption, then wage increases help combat corruption directly through the efficiency-wage argument, and indirectly by increasing the average motivation of public-sector workers.

## 2 Static Model

In this section we introduce a simple model that illustrates the relevant results.

### INSTITUTIONS AND WORKERS:

There are two institutions in the market, labeled  $A$  and  $B$ . The analysis focuses on the collective reputation and workforce composition of institution  $A$ , which is mission-oriented, while institution  $B$  is conceptualized as an outside option employment in a competitive market that is available to all workers.

There is a continuum of workers of measure one with a compact index set  $I$ . Workers are one of two types: Non-motivated or Motivated. Take  $m_i = 1$  if worker  $i$  is motivated and  $m_i = 0$  if non-motivated. A proportion  $\lambda$  of workers are motivated. Workers each have institution-specific abilities:  $y_i$  for institution  $A$  and  $x_i$  for institution  $B$ . For simplicity, we constrain  $y_i = 1$  for the main analysis (this assumption is relaxed in Section 5.2), while  $x_i$  is heterogenous and distributed according to a uniform distribution with support  $[\underline{x}, \bar{x}]$ . That is, all agents have same inherent ability at the mission-oriented institution, but vary in their outside option employment opportunity. Additionally,  $x_i$  is uncorrelated with worker motivation, although we also relax this assumption in Section 5.2.

Take  $a_i = 1$  if worker  $i$  is employed in institution  $A$ , and  $a_i = 0$  if  $i$  is employed in institution  $B$ .

### PAYOFFS:

Institution  $A$  has a demand for labor of measure  $\nu \leq \min\{\lambda, (1 - \lambda)\}$ , and receives the

following the profit, or net social output, from each individual it hires:

$$\pi_i^A = \pi y_i + \beta \mathbb{1}(m_i = 1) - w_i$$

Where  $y_i$  is the ability of worker  $i$ ,  $\beta$  reflects the higher productivity of motivated workers at institution  $A$  (we discuss the foundation for this modeling feature below), and  $w_i$  is the wage paid to worker  $i$ . Institution  $A$  does not observe  $\pi_i^A$  directly, but aggregate profit,  $\pi^A = \int_I \pi_j^A a_i$ , is publicly observable. Since all workers produce the same revenue in expectation, we constrain the wage in institution  $A$  to be constant across workers,  $w^A$ .

We add one assumption regarding institution  $A$ 's profit function, relative to  $\nu$ . Take  $x'$  that solves:

$$(1 - \lambda) \frac{x' - \underline{x}}{\bar{x} - \underline{x}} = \nu. \quad (1)$$

### Assumption 1

*The output institution  $A$  earns from each non-motivated worker,  $\pi$ , is greater than  $\nu + x'$ .*

This assumption ensures that, holding average worker motivation constant, institution  $A$  always maximizes profit at a wage that insures full employment.

### Definition 1 (Collective Reputation)

*The collective reputation of institution  $A$  is equal to  $C = \int_I a_i m_i / \int_I a_i$ .*

We define the collective reputation as the proportion of motivated types in institution  $A$  rather than aggregate behavior within  $A$ ; however, the two are equivalent here since types perfectly correlate with behavior. Therefore, this definition is consistent with reputation payoffs that depend on type-composition (social signaling and homophily) or institutional behavior (identity). In other words, agents can infer the composition of types within institution  $A$  by observing  $A$ 's aggregate profit (performance). Since the the collective reputation of  $B$  is perfectly negatively correlated with  $C$ , we can interpret  $C$  as  $A$ 's reputation relative to  $B$ 's (we do not explicitly consider payoffs associated with the collective reputation of  $B$ ).

Firm  $B$  receives following the profit from each individual it hires:

$$\pi_i^B = x_i - w_i$$

Where  $w_i$  is the wage paid to worker  $i$ . The individual's ability,  $x_i$ , is perfectly observed by the private firm and the private market is fully competitive. Therefore, we simply set each worker's outside option wage,  $w_i$ , equal to  $x_i$ .

Non-motivated workers have a standard linear utility function over own income:

$$U_n(w_i) = w_i$$

Where  $w_i$  is  $i$ 's wage.

Motivated workers differ from non-motivated workers in three regards: (1) they are more productive if matched with institution  $A$ , (2) they may value the mission of firm  $A$ , and hence may receive a direct benefit of employment in firm  $A$  (as in Francois, 2000 and Besley and Ghatak, 2005), (3) they value the collective reputation (workforce composition) of the mission-oriented sector  $A$ , e.g. due to type signaling or a direct preference for workplace homogeneity. To reflect both (2) and (3), motivated workers have a utility function of the following form:

$$U_m(w_i, C) = w_i + v(C)a_i$$

Where  $C$  is the proportion of motivated workers in institution  $A$ , and  $v(C)$  captures motivated workers payoffs from both collective reputation and mission;  $v(\cdot)$  is strictly increasing and concave.

To be consistent with the intuition that motivated workers place a positive value on a high reputation, and a negative value on a low reputation, we restrict the analysis to  $v(1) > 0$  and  $v(0) < 0$ . Additionally, the main analysis focuses on the case of  $v(\lambda) > 0$ . Given a “generic” firm, it may be natural to assume that, holding constant wage and reputation, a motivated agent perceives employment in firm  $B$  and employment in firm  $A$  as equivalent (which translates into  $v(\lambda) = 0$ ). However, the main focus of the paper is on “mission-oriented” institutions, where motivated agents are directly motivated by the mission, or product, of institution  $A$  (as in Besley and Ghatak 2005). In our model, given the constant product produced by each worker in institution  $A$ , mission-motivation simply translates into a constant benefit of working for institution  $A$ . Therefore, holding constant reputation and wage between sectors, motivated workers prefer working at the mission-oriented sector, which implies  $v(\lambda) > 0$ , where  $v(\lambda)$  represents the mission-benefits a motivated worker receives from employment in the mission-oriented institution.<sup>7</sup>

Since we are considering the wage of institution  $A$  as a policy tool, it is necessary to specify the framework for employment in institution  $A$  when it is over-demanded (i.e. demand for employment is greater than  $\nu$ ). Since all workers are ex-ante identical from  $A$ 's perspective, workers are randomly selected for employment in institution  $A$  from among the applicants.

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<sup>7</sup>In Section 5.1, we characterize the case of a generic firm ( $v(\lambda) = 0$ ) and show that mission-orientation is a necessary condition for reform.

Formally, workers choose  $\hat{a}_i \in \{0, 1\}$  at the beginning of the period, which determines employment according to the following rule:

$$a_i \begin{cases} = 0 & \text{if } \hat{a}_i = 0 \\ = 1 \text{ w.p. } q & \text{if } \hat{a}_i = 1. \end{cases}$$

Where:

$$q = \min \left\{ 1, \frac{\nu}{\int_I \hat{a}_i} \right\}.$$

#### OPTIMALITY AND EQUILIBRIUM:

Throughout the paper, we consider the objective of maximizing the efficiency of the mission-oriented sector, represented by social output,  $\pi^A$ .<sup>8</sup> Therefore, similar to the optimal-contracting approach of Besley and Ghatak (2005), we define outcomes given a wage in the mission-oriented institution, rather than analyzing  $w^A$  as an equilibrium choice.

Since information is complete, the equilibrium concept we use is Nash. That is, an equilibrium is defined by a set of employment choices,  $\{\hat{a}_i\}$ , such that given  $w^A$  and  $C$ , non-motivated workers set  $\hat{a}_i = 1$  iff  $w^A \geq x_i$ , non-motivated workers set  $\hat{a}_i = 1$  iff  $U_m(w^A, C, a_i = 1) \geq x_i$ , and  $C = \int_I \hat{a}_i m_i / \int_I \hat{a}_i$ .<sup>9</sup>

### 3 Analysis of the Static Model

In the analysis, we will use the terminology High/Low-Motivation to classify equilibria:

#### **Definition 2 (Collective Reputation High/Low-Motivation)**

*The collective reputation of institution A is High-Motivation if  $C > \lambda$  and Low-Motivation if  $C \leq \lambda$ .*

That is, the institution has a high-motivation reputation if a higher proportion of motivated workers are employed in institution A, relative to the population average. Additionally, we refer to an equilibrium with a high-motivation (low-motivation) reputation as a high-motivation (low-motivation) equilibrium.

First, we characterize equilibria in the static model in terms of cutoff types  $x_m$  for motivated workers, and  $x_n$  for non-motivated workers:

<sup>8</sup>However, note that given full employment in the mission-oriented sector, equilibria with greater  $\pi^A$  (higher  $C$ ) also produce greater aggregate utility, since both output in the mission-oriented institution and workers' reputation-payoffs are higher.

<sup>9</sup>A more precise definition of equilibrium would include the set of employment outcomes,  $\{a_i\}$ ; however, to allow for more efficient notation, we use the fact that with a continuum of workers,  $C$  can be defined as a function of  $\hat{a}_i$ .

**Lemma 1 (Cutoff Equilibrium)**

Given  $w^A$ , equilibrium employment decisions are characterized by  $\{x_m, x_n\}$ , where  $\hat{a}_i = 1$  if and only if  $m_i = 1$  and  $x_i \leq x_m$  or  $m_i = 0$  and  $x_i \leq x_n$ .

Lemma 1 states that, in equilibrium, conditional on type, individuals with a relatively low outside option will select into institution  $A$ . The result follows from single-crossing, by motivation-type, of  $u(x_i, a_i = 1)$  and  $u(x_i, a_i = 0)$  in  $x_i$ . (Formal proofs of all results can be found in Appendix A.)

Lemma 1 also allows us to characterize equilibria by identifying the private-sector abilities of individuals who are indifferent between the mission-oriented institution and institution  $B$ , and the corresponding reputation. That is, an interior equilibrium is defined by  $\{x_m, x_n, C\}$  that solve the following system of equations:

$$\begin{aligned} x_m &= w^A + v(C), \\ x_n &= w^A, \\ C &= \frac{\lambda(x_m - \underline{x})}{(1 - \lambda)(x_n - \underline{x}) + \lambda(x_m - \underline{x})}. \end{aligned}$$

Additionally, we define the function  $C(x, w^A)$  as:

$$C(x, w^A) = \frac{\lambda(x - \underline{x})}{(1 - \lambda)(w^A - \underline{x}) + \lambda(x - \underline{x})},$$

which allows us to define interior equilibria as  $\{x_m, x_n, C(x_m, w^A)\}$  such that  $x_m = w + v(C(x_m, w^A))$ . Moreover, this allows us to define  $u_m(x_i, C(x_i, w^A), a_i = 1)$ , which is the utility a motivated type,  $i$ , receives from employment in  $A$  if all motivated workers with  $x_j \leq x_i$  and all non-motivated workers with  $x_j \leq w^A$  are employed in  $A$ , and all others take their outside option. Since we define equilibria given  $w^A$ , when convenient we refer to  $C(x, w^A)$  as a function of  $x_m$  only ( $C(x_m)$ ). Note that an interior equilibrium can be characterized as a cutoff ability,  $x_i = x_m$ , that satisfies:

$$u_m(x_i, C(x_i), a_i = 1) = u_m(x_i, C(x_i), a_i = 0).$$

Given these definitions, we show that a high-motivation equilibrium exists for all  $w$ .

**Lemma 2 (Existence high-motivation equilibrium)**

Given  $w^A \in (\underline{x}, \bar{x})$ , there exists a unique high-motivation equilibrium,  $\{x_m, x_n, C^h\}$ , with  $C^h > \lambda$ .

Existence follows from mission-motivation: since  $v(\lambda) > 0$ , this implies that, given  $x_n = w$ , either an interior crossing of  $u_m(x_m, C(x_m), a_i = 1)$  and  $u_m(x_m, C(x_m), a_i = 0)$

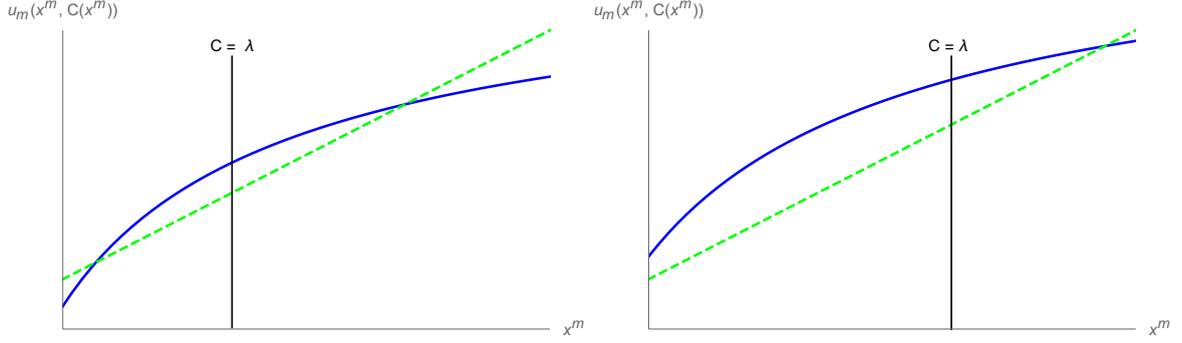


Figure 1: These graphs illustrate the respective utility of employment in  $A$  (blue, solid line) and the outside option (green, dashed line) for a motivated type with  $x_i = x_m$ , given that all motivated workers with  $x_i < x_m$  set  $\hat{a}_i = 1$ . The left graph has  $w^A < \underline{x} - v(0)$ , and the right graph has  $w^A > \underline{x} - v(0)$ .

or a corner equilibrium with  $x_m = \bar{x}$  exists. (For the existence of high-motivation equilibria when  $v(\lambda) \leq 0$ , see Section 5.1.) Since both  $v(\cdot)$  and  $C(x_m)$  are concave,  $u_m(x_m, C(x_m), a_i = 1)$  is concave in  $x_m$  while  $u_m(x_m, C(x_m), a_i = 0)$  is linear in  $x_m$ , which implies uniqueness.

Figure 1 illustrates equilibria for a given value of  $w^A$ ; the graph illustrates the respective utility of employment in  $A$  and the outside option for a motivated type with  $x_i = x_m$ , given that all motivated workers with  $x_i < x_m$  set  $\hat{a}_i = 1$ . Therefore, given  $x_n = w^A$ ,  $C(x_m)$  is increasing with  $x_m$ . Again, interior equilibria are represented by intersections of  $u_m(x_m, C(x_m), a_i = 1)$  and  $u_m(x_m, C(x_m), a_i = 0)$  since at an intersection, motivated workers with  $x_i = x_m$  are indifferent between employment in institution  $A$  and their outside option.

**Proposition 1 (Condition for Unique Equilibrium/Multiple Equilibria)**

If  $w^A > \underline{x} - v(0)$ , then the high-motivation equilibrium is the unique equilibrium of the model. If  $w^A \in (\underline{x}, \underline{x} - v(0))$ , then both high-motivation and low-motivation equilibria exist.

A graphical proof of Proposition 1 follows from Figure 1: If  $w^A > \underline{x} - v(0)$ , then  $u_m(x_m, C(x_m), a_i = 1)$  is greater than  $u_m(x_m, C(x_m), a_i = 0)$  at  $x_m = \underline{x}$ , and by the concavity of  $u_m(x_m, C(x_m), a_i = 1)$  in  $x_m$ , any intersection of  $u_m(x_m, C(x_m), a_i = 1)$  and  $u_m(x_m, C(x_m), a_i = 0)$  occurs in the range  $(\lambda, \bar{x}]$  (right graph). If  $w^A < \underline{x} - v(0)$ , however, then  $u_m(x_m, C(x_m), a_i = 1)$  is smaller than  $u_m(x_m, C(x_m), a_i = 0)$  at  $x_m = \underline{x}$ , which implies that an intersection of  $u_m(x_m, C(x_m), a_i = 1)$  and  $u_m(x_m, C(x_m), a_i = 0)$  occurs in the range  $(\underline{x}, \lambda)$  (left graph).

While Proposition 1 characterizes the existence of high/low equilibria as a function of the wage in institution  $A$ , it is of limited relevance when considering the optimal equilibrium from the perspective of institution  $A$ . Instead, the existence of high and

low-motivation market-clearing equilibria, where  $\int_I \hat{a}_i = \nu$ , will be a relevant benchmark: by Assumption 1, given a fixed level of average motivation, a market-clearing equilibrium is optimal from the perspective of the mission-oriented institution.

**Proposition 2 (Market-Clearing Equilibria)**

- (i) A high-motivation, market-clearing equilibrium  $\{w^h, C^{h'}\}$  exists.
- (ii) If  $\nu < (1 - \lambda)(\underline{x} - v(0))$ , then:

1. A low-motivation market-clearing equilibrium  $\{w^l, C^{l'}\}$  exists with  $w^l > w^h$ .
2. A low-motivation equilibrium exists for all  $w \in [\underline{x}, w^l]$  (including  $w^h$ ).

The existence of a high-motivation, market-clearing equilibrium follows from the fact that the cutoff abilities in the unique high-motivation equilibrium,  $\{x_m, x_n\}^h$ , are both continuous functions of  $w$ , which implies that a crossing with  $\int_I \hat{a}_i = (1 - \lambda)(x_n - \underline{x}) + \lambda(x_m - \underline{x}) = \nu$  exists. Proposition 2 also illustrates that both high and low-motivation market-clearing equilibria exist when institution  $A$ 's demand for labor is small relative to the overall labor market, and when the motivated type places a high valuation on reputation ( $|v(0)|$  is high).

Additionally, given the existence of a low-motivation market-clearing equilibrium, a low-motivation equilibrium exists for all wages in  $[\underline{x}, w^l]$ . This implies that, as long as the high-motivation market-clearing is an interior equilibrium, it is not the unique equilibrium for  $w = w^h$ . Therefore, as we discuss in the next subsection, achieving the optimal equilibrium is not always simply a matter of setting the optimal wage.

### 3.1 Optimal Contract

Next, we will partially characterize the optimal equilibrium from the point of view of the mission-oriented institution. Note that even though a market-clearing wage is optimal given a fixed level of average motivation (by Assumption 1), as the following proposition illustrates, the equilibrium that maximizes  $\pi^A(x_m, x_n, w^A)$  need not coincide with a market-clearing equilibrium.

**Proposition 3 (Optimal Equilibrium)**

*The equilibrium that maximizes  $\pi^A(x_m, x_n, w^A)$ ,  $\{w^*, C^*\}$ , is either equal to a market-clearing, high-motivation equilibrium,  $\{w^h, C^{h'}\}$ , or satisfies  $w^* \leq w^h$ ,  $C^* \geq C^{h'}$ .*

First, note that Proposition 3 implies that the optimal equilibrium must be a high-motivation equilibrium. This follows from the fact that for any interior low-motivation equilibrium, there exists an equivalent high-motivation equilibrium (with the same level of  $\int_I \hat{a}_i$ ) with higher net social output. Second, Proposition 3 states that  $\{w^*, C^*\}$  need not correspond to a market-clearing equilibrium: since full employment is optimal for  $A$  given a fixed level of reputation,  $w^* < w^h$  only if  $\int_I \hat{a}_i^* m_i > \int_I \hat{a}_i^h m_i$ . Formally, it

is possible for  $w^* < w^{A^h}$  since  $x_m$  can be a decreasing function of  $w$  – therefore, an equilibrium with a below-market wage may be optimal for  $A$ , since it may feature a greater *absolute* number of motivated workers.

While Proposition 3 identifies the optimal equilibrium, the existence of multiple equilibria implies that the model is indeterminate regarding the optimal contract. Specifically, Proposition 2 shows that both high and low-motivation equilibria exist for  $w^*$  when  $|v(0)|$  is large, which implies that the mission-oriented institution cannot ensure optimality by simply setting wages equal to  $w^*$ . Moreover, Proposition 2 suggests that institutions that simply set wages at market-clearing levels could find themselves with either a low or high level of motivation, depending on the underlying beliefs of agents as to which equilibrium the market will settle on (for a detailed discussion of the indeterminacy of policy analysis in the presence of multiple equilibria, see Morris and Shin, 2000).

It is also important to note that while Proposition 1 shows that the mission-oriented institution can always solve the problem of equilibrium multiplicity by setting a sufficiently high wage, such a solution is costly, since it requires a wage higher than  $w^*$ . In fact, depending on parameter values, setting a wage high enough to ensure the high-motivation equilibrium is unique may leave the mission-oriented institution *worse off* than in the low-motivation market clearing equilibrium. Therefore, instead of relying on criteria for equilibrium selection, we instead consider the problem of transitioning from a point of low-motivation to the optimal high-motivation equilibrium. Arguably, characterizing such a transition is a first-order concern, since the persistence of collective reputation (see Tirole, 1996) makes it unlikely that institutions in a high-motivation equilibrium will suddenly shift to a low-motivation equilibrium. However, the persistence of collective reputation, combined with legal and transactional constraints that prevent institutions from immediately replacing their entire workforce, also implies that moving from a low-motivation reputation to the optimal high-motivation equilibrium is likely to require a transition. Therefore, in the next section, we take a similar approach to Tirole (1996) and Acemoglu and Jackson (2014) and introduce a dynamic version of the model that accounts for frictions stemming from persistence in reputations, and that allows us to characterize a path for reforming a mission-oriented institution with a culture of low-motivation.

## 4 Dynamic Model

We now add a dynamic layer to the static framework, and consider a discrete-time dynamic framework with an infinite time horizon. We also introduce two plausible sources of friction to the dynamic model. The first and most important source of

friction is that motivated workers value the *lagged* collective reputation of institution  $A$ . This captures the notion that reputations and reputation payoffs are sticky, as perceptions might not update automatically (similar to Besley et al., 2014, as argued Jean Tirole (1996) “...stereotypes are long-lasting because new members of a group at least partially inherit the collective reputation of their elders”). Alternatively, this assumption serves as an approximation of a continuous-time model, where only atomless groups of workers join institution  $A$  at any moment. Crucially, this friction implies that transitions from low to high-reputation illustrated in this paper are not achieved through the assumption of coordinated action of a mass of motivated workers.<sup>10</sup>

Formally, workers have period-utility payoffs analogous to the static framework, with the exception that  $v(\cdot)$  is a function of  $(C_{t-1})$ :

$$U_m^t(w_i, C) = w_{i,t} + v(C_{t-1})a_{i,t}$$

Therefore, motivated workers’ period payoffs are not a function of their expectations regarding the proportion of motivated workers that will enter firm  $A$ ’s workforce in the *current* period. However, since expectations over *future* periods enter dynamic payoffs, the equilibrium path of  $\{C_t\}$  is not independent of expectations.

The second source of friction we introduce is that workers have employment tenure in institution  $A$ , in the sense that workers cannot be replaced by  $A$  directly. This assumption is of secondary importance (we characterize results when this assumption is non-binding), but increases the verisimilitude of the model to the underlying setting we consider, since it is unlikely that institutions are able to fire and replace all workers in a single period. Formally, this assumption introduces the possibility that employment in institution  $A$  has an option value. We will clearly detail when and how results are sensitive to this second feature of the dynamic model.

We also incorporate an exogenous method for replacement: a measure  $\delta \in (0, 1]$  of workers are “replaced” in each period, which we interpret as a natural rate of turnover due to, for example, retirement. Workers have an equal probability of being replaced, and are replaced by an individual of the same type and ability ( $\{m_i, x_i\}$ ). Importantly, replaced workers do not have employment tenure ( $a_{i,t-1} = 0$  for replacement workers). Therefore,  $\delta$  both functions as a discount rate, and ensures a minimum level of worker turnover in institution  $A$  ( $\int_I a_i \delta$ ). Additionally, workers are always free to voluntarily exit employment in the mission-oriented institution and take up employment in  $B$ .

Formally, as before, workers choose  $\hat{a}_{i,t} \in \{0, 1\}$  at the beginning of each period, and employment is determined according to the following rule, which also incorporates

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<sup>10</sup>Note that a simple repetition of the static game would produce the same equilibria as in the static game. Alternatively, an overlapping-generations approach could be used to similar effect, as in Acemoglu and Jackson (2014) and Acemoglu and Jackson (2015).

tenure:

$$a_{i,t} \begin{cases} = 0 & \text{if } \hat{a}_{i,t} = 0 \\ = 1 & \text{if } \hat{a}_{i,t} = 1, a_{i,t-1} = 1 \\ = 1 \text{ w.p. } q & \text{if } \hat{a}_{i,t} = 1, a_{i,t-1} = 0 \end{cases}$$

Where:

$$q = \min \left\{ 1, \frac{\delta\nu + \int_I \{\hat{a}_{i,t} = 0, a_{i,t-1} = 1\}}{\int_I \{\hat{a}_{i,t} = 1, a_{i,t-1} = 0\}} \right\}.$$

That is, if the mission-oriented institution is over-demanded, “open” slots in  $A$  ( $\delta\nu + \int_I \{\hat{a}_{i,t} = 0, a_{i,t-1} = 1\}$ ) are randomly allocated to new applicants ( $\int_I \{\hat{a}_{i,t} = 1, a_{i,t-1} = 0\}$ ). Additionally,  $C_t = C_{t-1}$  if employment in  $A$  is zero in time  $t$ .<sup>11</sup>

Institution  $A$  sets wages for all periods,  $\{w_t^A\}_1^\infty$ , at the beginning of period 1. The assumption that  $A$  can fully commit to wages is extreme – however, we will show that our results regarding a transition does not depend on this assumption. The timing of the period game is as follows:

1.  $A$  sets  $\{w_t^A\}_1^\infty$  (period 1 only).
2.  $\{w_t^A\}_1^\infty, C_{t-1}$  observed by workers.
3. Workers choose  $\hat{a}_{i,t} \in \{0, 1\}$ .
4. Period utility ( $a_{i,t}$ ) realizes.
5.  $\delta$  workers replaced at random.

DYNAMIC PAYOFFS:

Institution  $A$  has period payoffs that are equivalent to the static model, and dynamic payoffs equal to:

$$\Pi^A = \sum_{t=1}^{\infty} (1 - \tau)^{t-1} \pi_t^A.$$

Where  $\tau \in (0, 1)$ . As above, institution  $B$  sets wages equal to  $x_i$ .

For workers, the dynamic setting introduces the possibility of a positive option value of employment in institution  $A$ . Therefore, workers’ relative utility of employment in  $A$  takes the following form:

$$U^t(m_i, x_i, a_{i,t} = 1) = u_t(w_t^A, C_{t-1}, m_i) - x_i + (1 - \delta)O_i^t,$$

where  $u_t(w_t^A, C_{t-1}, m_i)$  is the period  $t$  payoff, and  $O_i^t$  represents the option value of employment in institution  $A$ :

$$O_i^t = (1 - q_{t+1}) [u(w_{t+1}^A, C_t, m_i) - x_i + (1 - \delta)O_i^{t+1}]$$

Note that the option value at  $t$  is non-zero only if  $q_{t+1} < 1$ ; that is, there is

<sup>11</sup>This rules out transition paths where institution  $A$  ‘resets’ its collective reputation by choosing a wage low enough such that employment is equal to zero.

no positive option value of holding a job in the mission-oriented institution unless  $A$  will be over-demanded in the following period. Therefore,  $O_i^t$  represents a sum of the expected benefit of holding a job in  $A$ , relative to applying in the following period, over a contiguous set of periods in which  $A$  is over-demanded.

Since we are concerned with the reform of an existing institution, we consider the situation where  $A$  “inherits” a reputation and workforce; that is, institution  $A$  is endowed with reputation  $C_0$ , and a  $t = 0$  workforce such that  $\int_I a_{i,0} = \nu$  and  $a_{i,0} = 1$  iff  $m_i = 1, x_i < x^m$  and  $a_{i,0} = 0$  iff  $m_i = 0, x_i < x^n$ .

#### EQUILIBRIUM:

Since information is complete, the equilibrium concept we use is sub-game perfect Nash Equilibrium. Additionally, we follow the selection-criterion of Gul et al. (1986) and assume that agents do not condition their choices on the past actions of sets of agents of measure zero, which insures that unilateral deviations by a single worker do not affect the actions of the other workers. Given a set of wages  $\{w_t^A\}_1^\infty$  and an initial reputation  $C_0$ , an equilibrium constitutes a set of employment choices,  $\{\hat{a}_{i,t}\}_1^\infty$ , that maximize each worker’s dynamic utility:

$$U^t(w_t^A, \{\hat{a}_{i,t}\}, C_{t-1}, m_i, x_i, \{w_t^A\}, \{q_t\}, \{C_t\}),$$

given the implied reputation,  $\{C_{t-1}\}_1^\infty$ , and demand for jobs at the mission-oriented institution,  $\{q_t\}_1^\infty$ .

Similar to the static section, we consider the objective of transitioning institution  $A$  to the steady-state equilibrium that maximizes net social output (we define steady-state equilibria formally below). While this objective is not always equivalent to maximizing the discounted stream of profits, as long as institution  $A$  has a discount rate,  $\tau$ , that is low enough, then a wage path that transitions to the steady-state that maximizes social output will always be preferable to remaining in any other steady state. We address optimal transitions after introducing our main result.

## 4.1 Analysis of Dynamic Model

Before addressing the issue of transition, we introduce some general results.

First, we characterize an individual’s decision rule, fixing  $\{w_t^A\}$ ,  $\{q_t\}$ , and  $\{C_t\}$ . Each worker chooses  $\hat{a}_{i,t} = 1$  if, and only if, the following expression holds:

$$u_t(w_t^A, C_{t-1}, m_i) + (1 - \delta)O_i^t \geq x_i. \quad (2)$$

Note that the decision rule is independent of  $a_{i,t-1}$ , since the employment preference is independent of tenure.

Again, given  $\{w_t^A\}$ ,  $\{q_t\}$ ,  $\{C_t\}$ , define  $x_t^m$  and  $x_t^n$  to be the ability of, respectively, the motivated and non-motivated types that are indifferent between working in institutions  $A$  and  $B$ . That is,  $x_t^m$  and  $x_t^n$  solve:

$$u_t(w_t^A, C_{t-1}, m_i) + (1 - \delta)O_i^t = x_i.$$

The following lemma extends the result of Lemma 1 to the dynamic model:

**Lemma 3 (Cutoff Equilibrium)**

Given  $\{w^A\}$ , equilibrium employment decisions are characterized by  $\{x_t^m, x_t^n\}$ , where  $\hat{a}_i = 1$  if and only if  $m_i = 1$  and  $x_i \leq x_t^m$  or  $m_i = 0$  and  $x_i \leq x_t^n$ .

Lemma 1 extends to the dynamic model since, by motivation-type,  $O_i^t$  is a monotonically decreasing function of  $x_i$ . Lemma 3 allows us to characterize an equilibrium in terms of cutoff types  $x_t^m$  and  $x_t^n$ , analogous to the static case.

The main focus of our analysis is on the optimal steady-state equilibrium, where steady-state equilibria are defined as follows:

**Definition 3 (Steady-State Equilibria)**

Given  $\{w_t^A\}$  such that  $w_t^A = \bar{w}^A$  for all  $t$ , an equilibrium  $\{x_t^m, x_t^n, C_t\}_t^\infty$  is a steady-state equilibrium if  $x_t^m = \bar{x}^m$ ,  $x_t^n = \bar{x}^n$ , and  $C_t = \bar{C}$  for all  $t$ .

The relationship between static and dynamic equilibrium is clarified by the following proposition:

**Proposition 4 (Static Equilibrium  $\Leftrightarrow$  Steady-State Equilibrium)**

For each static equilibrium,  $\{w^A, x^m, x^n, C\}$ , there exists a corresponding steady-state equilibrium,  $\{\bar{w}^A, \bar{x}^m, \bar{x}^n, \bar{C}\}_t^\infty$ , with  $\bar{w}^A = w^A$ ,  $\bar{x}^m = x^m$ ,  $\bar{x}^n = x^n$ , and  $\bar{C} = C$ ; and for each steady-state equilibrium, there exists a corresponding static equilibrium.

Proposition 4 shows that when both high and low-reputation equilibria exist in the static model, then corresponding high and low-reputation steady-state equilibria exist in the dynamic model. Additionally, it gives the following corollary:

**Corollary 1**

The optimal steady-state equilibrium  $\{\bar{w}^{A*}, \bar{C}^*\}_t^\infty$ , corresponds to the optimal static equilibrium,  $\{w^{A*}, C^*\}$ .

In the following text, we use  $\{w^{A*}, C^*\}$  to refer to the optimal steady-state equilibrium. The following section analyzes the possibility of a dynamic transition from a low-reputation steady-state to  $\{w^{A*}, C^*\}$ , precipitated by a designer who sets wages in institution  $A$ ,  $\{w_t^A\}_1^\infty$ .

## 4.2 Dynamic Transition

To formalize the problem of transition introduced at the end of Section 3, we address the question of whether a wage path  $\{w_t^A\}_1^\infty$  exists that induces a transition in the state variable,  $C_t$ , from  $C_0 < \lambda$  to  $C_t = C^*$ , where  $\{w^{A*}, C^*\}$  is the steady-state equilibrium that maximizes  $\pi_t^A$ . While the analysis is general, we often refer to the case where  $C_0$  corresponds to a low-reputation market-clearing steady-state  $(\bar{C}_0, \bar{w}_0^A)$ , since this addresses the question of transition from a dynamic equilibrium corresponding to a low-reputation static equilibrium to a dynamic equilibrium corresponding to the optimal static equilibrium. In these cases, we refer to  $\bar{w}_0^A$  as the starting wage, and our description of a wage path includes  $\bar{w}_0^A$ .

### 4.2.1 Example: the case of $\delta = 1$

To facilitate the exposition of the dynamic results, we begin by characterizing transitional wage paths and establishing conditions for their existence given  $\delta = 1$ . With  $\delta = 1$  there is full replacement in each period, which implies that tenure is non-binding and agents' dynamic payoffs are equal to their period payoffs. This allows us to illustrate the main findings of the model in a relatively simple manner.

Since workers only live for a single period, there is no option value of employment in institution  $A$  and workers simply choose  $\hat{a}_{i,t}$  to maximize period utility:

$$U^t(C_{t-1}, m_i, x_i, w_t^A, \{q_t\}) \begin{cases} = x_i & \text{if } \hat{a}_{i,t} = 0 \\ = q_t(w_t^A + m_i v(C_{t-1})) + (1 - q_t)x_i & \text{if } \hat{a}_{i,t} = 1, \end{cases}$$

where  $q_t = \min\{1, \int_I \hat{a}_{i,t}/\nu\}$ . This implies the following lemma:

#### Lemma 4 (Uniqueness)

With  $\delta = 1$ , given  $C_{t-1}$  and  $w_t^A$ , there exists a unique period-equilibrium  $\{x_t^m, x_t^n, C_t\}$ . Additionally, given an interior  $\int_I \hat{a}_{i,t}$ ,  $A$ 's reputation in period  $t$ ,  $C_t$ , is high-motivation (low-motivated) if, and only if,  $v(C_{t-1}) > 0$  ( $v(C_{t-1}) \leq 0$ ).

Given uniqueness, we can characterize the comparative statics of the model with respect to wages.

#### Lemma 5 (Crowding out/in motivation)

If  $v(C_{t-1}) \leq 0$ , then  $\partial C_t(w_t^A)/\partial w_t^A \geq 0$ .

If  $v(C_{t-1}) > 0$ , then  $\partial C_t(w_t^A)/\partial w_t^A \leq 0$ .

Lemma 5 states that the direction of the effect of current-period wages on reputation depends only on whether the reputation payoff the motivated type receives from employment in institution  $A$  is positive or negative: if the reputation payoff is positive,

then higher wages crowd out motivated types; if the reputation payoff is negative, then higher wages *crowd in* motivated types.

The proof follows from the linearity of utility in the public sector wage. Intuitively, since  $x^m$  and  $x^n$  are linear functions of  $w_t^A$ , a wage increase effectively adds a mass of workers to institution  $A$  to who have an average motivation of  $\lambda$ , which moves  $A$ 's reputation closer to  $\lambda$ . And since, by Lemma 4,  $C_t \leq \lambda$  if and only if  $v(C_{t-1}) \leq 0$ , increasing  $w_t^A$  will increase  $C_t$  in this case, while the reverse is true for  $v(C_{t-1}) > 0$ .<sup>12</sup>

Lemma 5 also provides insight regarding potential transition paths between an initial steady-state with low reputation,  $\{\bar{w}_0^A, \bar{C}_0\}$ , and the optimal steady-state,  $\{w^{A*}, C^*\}$ :

**Lemma 6 (Non-Monotonic Transition)**

*Given an initial reputation,  $C_0$ , such that  $v(C_0) < 0$ , no monotonic wage path exists that transitions to  $\{w^{A*}, C^*\}$ .*

Lemma 6 follows from the comparative statics outlined in Lemma 5, and shows that given a low initial reputation, a transition cannot be achieved by a wage path that simply decreases  $w_t^A$  to  $w^{A*}$ .<sup>13</sup> Therefore, if a transition path exists from an initial low-motivation steady-state, it must be non-monotonic.

The next result shows that a wage path always exists that transitions between a low-motivation and high-motivation reputation, and characterizes the non-monotonic wage path that enables this transition.

**Proposition 5 (Existence of Transition)**

*Given  $\delta = 1$ , a wage path that transitions to  $\{w^{A*}, C^*\}$  exists for any  $C_0$ .*

We focus on the proof for the case where  $v(C_0) < 0$ , which also demonstrates the result for  $v(C_0) \geq 0$ . Additionally, we consider the case where the optimal steady-state equilibrium,  $\{w^{A*}, C^*\}$ , corresponds to a unique market-clearing steady-state equilibrium,  $\{w^{A^h}, C^{h'}\}$ . (Other cases are addressed in Appendix A.)

To illustrate the non-monotonic transition, we prove the result by construction. Specifically, the following non-monotonic wage path transitions from  $v(C_0) < 0$  to  $\{w^{A*}, C^*\}$ :

1.  $w_1^A$  solves  $w_1^A + v(C_0) = \bar{x}$  ( $w_1^A$  is set high enough that  $\hat{a}_{i,1} = 1$  for all  $i$ ).
2.  $w_t^A$  for  $t > 1$  solves  $\int_I \hat{a}_{i,t} = \nu$  (after period 1,  $w_t^A$  is set at the market-clearing level).

<sup>12</sup>Simply put, Lemma 5 states that wage increases moves the current-period reputation closer to that of the population average since the distance between  $x^m$  and  $x^n$  is fixed by  $v(C_{t-1})$ . Clearly this will not always be true locally for all distributions of  $x_i$ , however, the result holds more generally for changes in the wage that are large enough.

<sup>13</sup>Note that  $v(C_0) < 0$  implies  $C_0 < \lambda$ , but the reverse need not be true; a transition can be achieved with a monotonic wage path with  $C_0 < \lambda$  and  $v(C_0) > 0$  since, by Lemma 4,  $C_1$  will be greater than  $\lambda$ .

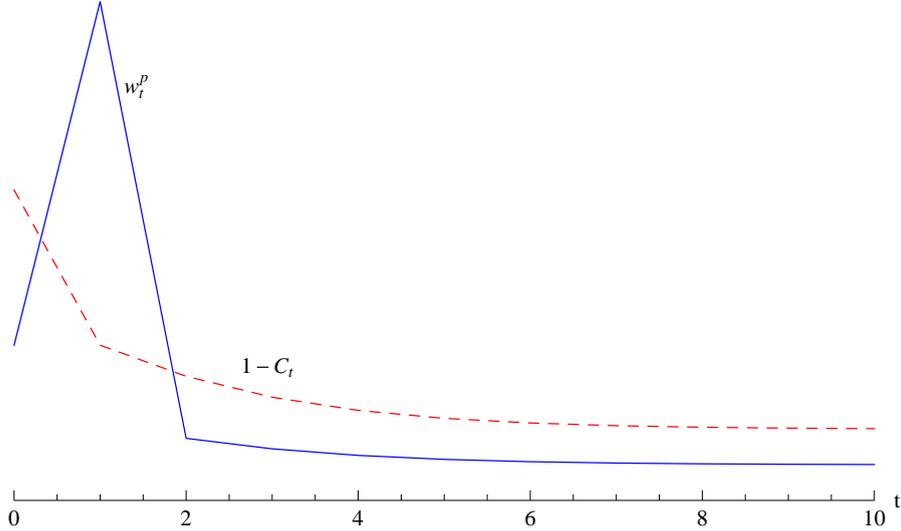


Figure 2: This graph illustrates a wage path that transitions from a high-corruption to a low-corruption equilibrium. Note the initial increase in the wage in the mission-oriented institution ( $w_t^A$ ; solid line), followed by a decrease and convergence to the efficiency wage. Corruption ( $(1 - C_t)$ ; dashed line), however, decreases monotonically.

To see why this wage path results in a transition, it is informative to solve for the market-clearing wage and the corresponding  $C_t$  in each period. Initially, institution  $A$  is endowed with low-motivation reputation,  $v(C_0) < 0$ , and by Lemma 5,  $A$ 's reputation can only be increased by increasing wages. Taken to the extreme,  $w_1^A$  is set at a high enough level such that all workers prefer employment in institution  $A$ , and  $A$ 's reputation in period 1 will replicate the population average ( $C_1 = \lambda$ ).

In period 2, given  $v(C_1 = \lambda) > 0$ , Lemma 5 states that  $C_2$  is weakly decreasing in wages, and Lemma 4 implies that  $C_2 > \lambda$  for  $\int_I \hat{a}_{i,t} = \nu$ . Moreover, the wage that solves  $\int_I \hat{a}_{i,2} = \nu$  is smaller than  $w_1^A$ , since at  $w_2^A = w_1^A$ , all agents would set  $\hat{a}_{i,2} = 1$ . Together, this implies that  $w_2^A < w_1^A$ ,  $C_2 > C_1$ .

In period 3, the market-clearing wage,  $w_3^A$  is strictly lower than  $w_2^A$ , since  $C_2 > C_1$  makes employment in institution  $A$  relatively more attractive for motivated workers ( $v(C_2) > v(C_1)$ ). Furthermore, by Lemma 5,  $w_3^A < w_2^A$  implies that  $C_3 > C_2$ .

By the same logic as in period 3, in all future periods, the market-clearing wage is decreasing and  $A$ 's reputation is increasing. Moreover, since the sequences  $\{w_t^A\}$  and  $\{C_t\}$  are bounded and monotonic,  $\{w_t^A, C_t\}$  must converge as  $n \rightarrow \infty$ . Lastly, the points of convergence must correspond to  $\{w^{A*}, C^*\}$ , since this is the unique market-clearing, high-motivation steady-state.

The transition outlined above illustrates the general shape of the non-monotonic path of wages (also illustrated visually in Figure 2). Starting from a point of low-motivation,  $w^A$  must be increased to induce motivated workers to join institution  $A$ ,

hence “purchasing” a higher reputation. Once a sufficiently high reputation has been reached – with  $\delta = 1$  this occurs in a single period – the process is reversed and public-sector wages are lowered, disproportionately driving non-motivated workers out of the public sector.

Note that this is not the unique transition path, but it ensures full employment during the transition. Other wage paths can converge to  $\{w^{A*}, C^*\}$  in finite time:  $C_2$  can be set arbitrarily close to 1 by decreasing  $w_2^A$  below the market-clearing level. Therefore, there exists a  $w_2^{A'}$  such that  $C_2 = C^*$ , and the high-reputation stable point is reached in period 3. (We discuss optimal transitions below.)

#### 4.2.2 General analysis: $\delta \in (0, 1)$

The intuition from the example with  $\delta = 1$  carries over to the more general model. In particular, the following proposition partially characterizes the existence of a transition from a low-motivation point to the optimal steady-state equilibrium:

**Proposition 6 (Existence of transition  $v(C_0) < 0 \Rightarrow \{w^{A*}, C^*\}$ )**

*If  $v(C_0) < 0$ , then there exists  $\{w_t^A\}'$  such that an equilibrium with  $w_t^A \rightarrow w^{A*}$  and  $C_t \rightarrow C^*$  exists.*

Proposition 6 shows that an equilibrium transition is always possible for a mission-oriented institution – an equilibrium exists that transitions to  $\{w^{A*}, C^*\}$  that is analogous to the example given for  $\delta = 1$ . Unlike the case with full replacement, however, since  $\delta < 1$ , a shift to  $C_t = \lambda$  cannot be achieved in a single period: given  $w^A$  high enough that all workers set  $\hat{a}_i = 1$ ,  $A$ 's reputation will increase slowly as only a measure of  $\delta\nu$  tenured workers in  $A$  are replaced in each period. However, after  $C_{t-1}$  reaches a threshold level where  $v(C_{t-1}) > 0$  (this level can always be reached through a wage increase since  $C_t \rightarrow \lambda$  and  $v(\lambda) > 0$ ), then the transition to  $\{w^{A*}, C^*\}$  can be achieved by a market-clearing wage path.<sup>14</sup>

However, this result merely illustrates that there exists a wage path and a corresponding equilibrium that transitions – multiple equilibria may exist for any wage path. Specifically, for the “market-clearing” wage path described above, the equilibrium is unique over the initial periods since, given the high wage, all workers set  $\hat{a}_i = 1$ , but for the set of periods in which the wage is decreasing, multiple equilibrium outcomes are possible. The following result extends Proposition 6 to all equilibria:

**Proposition 7 ( $v(\lambda) > 0$ : Transition in all Equilibria)**

*If  $v(C_0) < 0$ , then there exists  $\{w_t^A\}'$  such that in any equilibrium,  $w_t^A \rightarrow w^{A*}$  and  $C_t = C^* + \epsilon$  or  $C_t \rightarrow C^* + \epsilon$  for some  $\epsilon \geq 0$ .*

<sup>14</sup>Note that this transition path does not rely on the assumption of commitment to a wage path: analogous to the case of full replacement, along the whole path of transition workers' employment choices are consistent with the choice that maximizes their period utility.

Proposition 7 shows that while multiple equilibria may exist for a transition wage path, in all equilibria  $\{w_t^A\}$  leads to a transition to some  $C' \geq C^*$ . The intuition can be illustrated using the “market-clearing” wage-path illustrated above: while multiple equilibria exist for the subset of  $\{w_t^A\}$  where the wage is decreasing,  $x_t^m$  remains unique, which implies that other equilibria can only occur when  $x_t^m$  is greater than the market-clearing level. This implies that in all equilibria, the proportion of motivated workers in institution  $A$  is weakly greater than under the market-clearing equilibrium, which gives the result that in all equilibria,  $C_t$  is weakly greater than the reputation in the market-clearing equilibrium.

### 4.2.3 Optimal Transition Path

Many public and other mission-oriented institutions may face legal and budgetary constraints that limit their ability to raise wages in any given period. Therefore, we detail the transition path that minimizes the maximum wage bill of institution  $A$ .<sup>15</sup> To avoid an open-set problem, we assume that wages can only be increased in multiples of a discrete unit,  $\epsilon$ , where  $\epsilon$  is arbitrarily small:

#### Proposition 8 (Minimum Budget)

*The following wage path minimizes the maximum budget required to transition from any  $C_0$  such that  $v(C_{t-1}) \leq 0$  to  $\{w^{A^*}, C^*\}$ :*

$$w_t^A = \begin{cases} \underline{x} + v(0) + \epsilon & \text{for } t \text{ s.t. } v(C_{t-1}) \leq 0 \\ w^{A'} \text{ where } w^{A'} \text{ solves } C_t = C^* & \text{for } t' = \min\{t | v(C_{t-1}) > 0\} \\ w^{A^*} & \text{for } t > t'. \end{cases}$$

The proof of Proposition 8 can be demonstrated simply using the best-response dynamics of the static model since, given a fixed wage, an equilibrium of the dynamic model exists where the period-equilibrium converges to a stable equilibrium of the static model. Moreover, the only static equilibria that are stable are either corner solutions, or equilibria where  $U_m(x^m, a_i = 1)$  crosses  $U_m(x^m, a_i = 0)$  from above. This implies that, given  $w_t^A = \underline{x} + v(0)$ , which ensures that  $x^m = \underline{x}$  is not an equilibrium, the unique stable static equilibrium is a high-motivation equilibrium, and the dynamic model will converge to a high-motivation point. In other words, if the wage is set high enough that the motivated worker with the lowest ability wishes to join the public sector even when  $C = 0$ , then there is a unique stable equilibrium of the static model at  $C' > \lambda$ . Therefore, regardless of  $C_0$ , the dynamic model will converge to  $C'$ , and  $t'$

<sup>15</sup>The “utility-optimal” transition path is not well-defined, given that multiple equilibria exist for most wage paths that result in a transition. Even disregarding multiplicity, a characterization of a transition that maximizes  $A$ ’s utility will be highly sensitive to the precise tradeoff between the speed of the transition and its aggregate cost implied by the discount rate and other parameters.

such that  $v(C_{t-1}) > 0$  will be reached in finite time.

Proposition 8 also shows that as soon as  $t'$  such that  $v(C_{t'-1}) > 0$  is reached, then a transition to the efficient steady-state can be achieved in a single period by setting  $w_{t'}^A$  low enough that institution A is under-demanded. This transition path implies that institution A will not achieve a maximum profit in period  $t'$ . However, this may still be a profitable strategy, since it achieves a faster transition to the efficient steady-state equilibrium.<sup>16</sup>

Lastly, we point out that a transition can be achieved even if the mission-oriented institution is legally constrained to keep the wage of past employees constant or non-decreasing. As long as institution A can change  $w^A$  for incoming cohorts, then the wage paths described above will still result in a transition. In this case, however, a transition can only be achieved through the natural rate of replacement, since non-motivated workers cannot be actively pushed out of their jobs through lower wages.

## 5 Extensions

Here we discuss several relevant extensions.

### 5.1 Transition in Generic Firms ( $v(\lambda) = 0$ )

In this section, we show that  $v(\lambda) > 0$  is not only a sufficient condition for a wage path that transitions to  $\{w^{A*}, C^*\}$  in all equilibria, it is also a necessary condition. Specifically, we consider the case of a “generic” firm without any mission-payoffs; i.e. where motivated workers are indifferent between institutions A and B when  $w^A = x_i$  and  $C = \lambda$ , which translates to  $v(\lambda) = 0$ . (Results are analogous for  $v(\lambda) < 0$ .)

First, we state the analogous result to Proposition 2:

#### **Proposition 1' (Existence of Multiple Equilibria')**

*If  $v(\lambda) = 0$ , there exists a high-motivation equilibrium if  $\nu$  is small enough and  $v(1)$  large enough such that  $\nu < \lambda(\underline{x} + v(1))$ , and there exists a low-motivation equilibrium.*

Similar to Proposition 2, Proposition 1' shows that when  $v(\lambda) = 0$ , multiple equilibria exist when institution A's demand for labor is relatively small compared to the overall labor market, and when the motivated type places a high valuation on reputation ( $v(1)$  is high).

Moving on to the dynamic model,

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<sup>16</sup>This strategy of transition may not be robust, since a large drop in salary may disturb the employer-employee relationship by, for example, eroding trust (see Sliwka, 2007 and Ellingsen and Johannesson, 2008). Therefore, we have highlighted transition paths that involve market-clearing wages.

**Proposition 4' (Non-Existence of Transition)**

Given  $\delta = 1$ , a wage path that transitions from  $v(C_0) < 0$  ( $C_0 < \lambda$ ) and  $\{w^{A*}, C^*\}$  does not exist if  $v(\lambda) = 0$ .

The intuition for the nonexistence result for  $v(\lambda) = 0$  follows from the same wage path illustrated in Section 4.1.1. Note that  $C_1 = \lambda$  can always be achieved by setting a high wage in the first period. However, in the second period, an increase in the proportion of motivated workers cannot be achieved through a wage decrease since  $v(C_1) = v(\lambda) = 0$ , and by Lemma 5,  $A$ 's reputation is weakly increasing in  $w^A$ .

Proposition 5 and Proposition 4' demonstrate that the existence of a transition path depends on whether a point such that  $v(C_{t-1}) > 0$  can be reached through a wage increase. If not, the region of  $C$  where the proportion of motivated workers is increasing in  $w^A$  cannot be reached, and a transitional wage path does not exist. In a generic institution,  $v(C_{t-1}) > 0$  only if  $C_{t-1} > \lambda$ , which implies that a point with  $v(C_{t-1}) > 0$  cannot be reached through a wage increase. In a mission-oriented institution, however, motivated workers strictly prefer working in the institution given a neutral reputation. Therefore,  $v(C_{t-1}) > 0$  can be achieved through a wage increase, which enables a transition that is unavailable to generic firms.

In the general case of  $\delta \in (0, 1)$ , a weaker result than Proposition 4' holds:

**Proposition 5' (Existence of transition to  $\{w^{A*}, C^*\}$ )**

If  $v(\lambda) = 0$  and  $v(C_0) < 0$ , then for any  $\{w^A\}$  there exists an equilibrium such that  $C_t \leq \lambda$  for all  $t$ .

That is, in contrast to the case of full replacement, Proposition 5' does not fully rule out the possibility of a transition when  $v(\lambda) = 0$  – under certain conditions, an expectations-driven transition can be achieved.

To illustrate the possibility of a expectations-driven transition, take the following example: Assume for simplicity that  $C_0 = \lambda$  (the population average can always be replicated through a wage increase) and that institution  $A$  *commits* to the following wage path:

1.  $w_1^A = w_2^A$  market-clearing given  $C = \lambda$ ,  $O_i^t = 0$ .
2.  $w_t^A$  market-clearing, given expectations that  $\int_I \hat{a}_i = \nu$ .

Now, suppose workers hold the belief that  $C_1 > \lambda$ , and hence expect that institution  $A$  will be over-demanded in period 2. In this case,  $O_i^1(m_i = 1, x'_i) > O_i^1(m_i = 0, x'_i)$ . This in turn implies that  $v(\lambda) + O_i^1(m_i = 1) > O_i^1(m_i = 0)$ , and  $C_1 > \lambda$ .

That is, given the expectation that  $C_1$  will be greater than  $\lambda$ , and that institution  $A$  will be over-demanded in period 2, the option value of holding a job in  $A$  in period 1 is higher for motivated types than non-motivated types. This implies that motivated workers will disproportionately enter into institution  $A$  in period 1, making the belief

that  $C_1$  will be greater than  $\lambda$  self-fulfilling. After period 2, given that  $C_t$  is greater than  $\lambda$ , the wage path detailed above will transition to a low-corruption equilibrium by the same logic as the proof of Proposition 6.

Note that an expectations-driven transition requires both that workers hold “optimistic” beliefs regarding the future reputation of  $A$ , *and* that institution  $A$  is able to commit to holding wages above a market-clearing level even after its reputation has increased above  $\lambda$ . Absent commitment,  $A$  would prefer to set wages at a market-clearing level in period 2; however, this would imply that  $O_t^1 = 0$ , which would destroy the incentive for motivated workers to disproportionately enter institution  $A$  in period 1. That is, in contrast to the case of a mission-oriented institution where a transition does not depend on workers’ expectations, in a generic institution, absent commitment to future wages, the expectations-driven transition would unravel.

## 5.2 Exploiting Correlation between Ability and Motivation

It is natural to imagine that an agent’s ability and their level of motivation may be correlated. For example, due to either selection or socialization, it is often suggested that individuals with a degree in economics are less prosocial and, depending on the mission of their workplace, may therefore be less motivated. Here, we consider the case where ability in institution  $A$  is heterogenous and correlated with motivation, and show that this extension of the model suggests potentially important insights for transitioning to a high-motivation steady state.

We amend the baseline model by introducing heterogeneity in worker ability within institution  $A$ ,  $y_i$ . Specifically,  $y_i \in \{y_1, \dots, y_p\}$  and, for simplicity, we assume that there is a measure  $1/p$  of each ability-type with compact index set  $I^p$ . Additionally, a proportion  $\lambda^p$  of each ability-type is motivated (abusing notation, we use a  $p$  superscript to refer to variables that are differentiated by ability in institution  $A$ ); take  $\bar{\lambda}$  to be the average level of motivation of the population,  $\bar{\lambda} = \sum^p \lambda^p/p$ . To introduce correlation between ability and type, we assume that  $\lambda^i \neq \lambda^j$  for some  $i, j \in \{1, \dots, p\}$ .

We also allow for a correlation between  $y_i$  and  $x_i$ : given  $y_i = y_p$ , ability in institution  $B$  is uniformly distributed over  $[\underline{x}^p, \bar{x}^p]$ . Again, to focus on the problem of selecting based on motivation, ability  $(x_i, y_i)$  is observable and motivation is unobservable. Analogous to the model above, institution  $A$  has a unit demand of  $\nu^p < 1/p$  of each ability-type, and sets a uniform wage conditional on  $y_i$ ,  $w^{A,p}$ .

Lastly, take  $C_t^p$  to equal the average level of motivation by ability-level, and  $C_t$

equal to the average reputation of institution  $A$ :

$$C_t^p = \frac{\int_{I^p} m_i a_{i,t}}{\int_{I^p} a_{i,t}},$$

$$C_t = \frac{\sum^p \int_{I^p} m_i a_{i,t}}{\int_I a_{i,t}}.$$

The following proposition illustrates that, depending on the precise nature of reputation-payoffs, the correlation between ability and motivation can be exploited to transition between a low-motivation point and the high-motivation steady state equilibrium (the argument for existence of a high-motivation steady state equilibrium is analogous to the baseline model).

**Proposition 9 (Transition of Average Reputation)**

(i) *If motivated agents value ability-contingent reputation, i.e.  $U_{m,t}(w_i, C) = w_{i,t} + v(C_{t-1}^p)a_{i,t}$ , then a wage path that transitions from  $v(C_0) < 0$  to  $\{w^{A*}, C^*\}$  in all equilibria exists if, and only if,  $v(\lambda^p) > 0$ .*

(ii) *If motivated agents value the average reputation of the institution, i.e.  $U_{m,t}(w_i, C) = w_{i,t} + v(C_{t-1})a_{i,t}$ , then a wage path that transitions from  $v(C_0) < 0$  to  $\{w^{A*}, C^*\}$  in all equilibria exists if  $v(\bar{\lambda}) \geq 0$ .*

Result (i) is a straightforward corollary of Proposition 6: if motivated agents value ability-contingent reputation, then each ability category can be treated as its own institution. Result (ii), however, illustrates that if motivated agents value average reputation, then institution  $A$  can exploit the correlation between ability and motivation to transition to a high-motivation steady state even if  $v(\bar{\lambda}) = 0$ . This result follows from the simple intuition that  $A$  can manipulate its reputation by disproportionately hiring agents from ability levels with average levels of motivation above that of the population average (see Appendix A for the formal proof).

The insights from this section may inform a commonly-attempted strategy for reforming an institution by creating an “elite” division within the institution that is staffed by highly-motivated individuals. Logically, this strategy may be successful if recruitment for the elite division targets a group of individuals with a high average motivation, proxied by, say, a certain level educational achievement. However, given a poor institutional reputation, the institution faces an adverse selection problem that may cause this strategy may fail: among the target group, individuals with low motivation are disproportionately attracted to the elite division, since the low collective reputation of the institution dissuades high-motivation individuals from applying. Therefore, this strategy can only be successful if the elite division targets a group of individuals with a high average motivation *and* sets a wage high enough to overcome the adverse selection problem caused by the institution’s low collective reputation.

## 6 Conclusion

In this paper, we analyze a model of labor-market sorting with motivated agents who value both mission and collective reputation. We highlight the prediction that the effect of wages on motivation is conditional upon the collective reputation of the mission-oriented institution, which implies that a non-monotonic wage path is needed to transition from a low-motivation to a high-motivation steady-state.

While many factors certainly contribute to the emergence of Weberian public bureaucracies, it is informative to consider the path of public-sector wages during Sweden’s transition to a modern, well-functioning bureaucracy. Currently, Swedish bureaucrats enjoy a reputation as belonging to one of the internationally best-regarded systems of public administration and, in line with the predictions of a public-efficiency wage, these results are achieved with a workforce that is paid seven to fourteen percent less than peers in the private sector.<sup>17</sup> However, in the mid-1800’s corruption was endemic to the Swedish system public administration, where the system of payments to government officials involved “gifts” for services rendered (see Rothstein, 2011). It was only after a period of reform, involving a radical transformation of the system of payment, that the Swedish bureaucracy evolved into the efficient institution we see today. Interestingly, while public-sector wages were initially increased to ensure that bureaucrats were not dependent on direct payments from citizens, and to open the profession to a larger class of individuals, public-sector wages fell relative to private-sector wages in “the decades surrounding the Second World War” (Granholm, 2013 pp. 101, translated from Swedish). Moreover, Sundell (2013) provides evidence that nepotism in the Swedish bureaucracy decreased continually through this period, and argues that this is indicative of a continuous transition to a well-functioning public sector, a pattern that is consistent with the transition path we outline here.

We conclude with some comments on how the mechanism for transition we introduce here can complement other efforts at reform, and with avenues for future research. First, it is important to note that the non-pecuniary motives that we analyze depend on the composition of types in the public institution, rather than the precise level of corruption. That is, type-signaling and homophily are independent of the precise behavior of non-motivated and motivated types, as long as there is a difference in behavior between the two types that can be identified through the aggregate behavior of the institution. Therefore, a direct anti-corruption measure, such as improved monitoring, is orthogonal to our mechanism as long as workers update their expectations of each type’s behavior.

Additionally, our mechanism is complementary to efforts to change institutional

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<sup>17</sup>Controlling for observables, de Koning and de Hek, 2013 find an average differential of seven percent amongst central government workers, and fourteen percent for local government.

culture by changing institutional norms: If some proportion of workers are conformist (see Bernheim, 1994 and Huck et al., 2012 for models of social norms based on conformity), and hence switch from non-motivated to motivated given some threshold level of aggregate motivation, then increasing the proportion of motivated types in the institution due to selection will precipitate a complementary shift in behavior of the conformist types. This will in turn speed the transition by improving the institution's collective reputation.

Lastly, the model we present here is tailored to address the issue of reform in the mission-oriented sector. However, institutional reputation is also important for targeting talented and motivated individuals in the private sector (see Bhattacharya et al., 2008). And while we address reform of generic institutions in Section 5.1 in a partial-equilibrium setting, a relevant extension of our analysis would be to consider reputation in a setting with endogenous mission and competition over motivated workers.

## References

- Acemoglu, Daron and Matthew Jackson (2014), "Social Norms and the Enforcement of Laws." *NBER Working Paper Series*.
- Acemoglu, Daron and Matthew Jackson (2015), "History, Expectations, and Leadership in the Evolution of Social Norms." *The Review of Economic Studies*, 82, 423–456.
- Akerlof, George and Rachel Kranton (2000), "Economics and Identity." *The Quarterly Journal of Economics*, 115, 715–753.
- Akerlof, George and Rachel Kranton (2005), "Identity and the Economics of Organizations." *The Journal of Economic Perspectives*, 19, 9–32.
- Aldashev, Gani, Esteban Jaimovich, and Thierry Verdier (2014), "When warm glow burns: Motivational (mis)allocation in the non-profit sector." *Working Paper*.
- Ariely, Dan, Anat Bracha, and Stephan Meier (2009), "Doing Good or Doing Well? Image Motivation and Monetary Incentives in Behaving Prosocially." *The American Economic Review*, 99, 544–555.
- Auriol, Emmanuelle and Stefanie Brilon (2014), "Anti-social behavior in profit and nonprofit organizations." *Journal of Public Economics*, 117, 149–161.
- Banerjee, Ritwik, Tushi Baul, and Tanya Rosenblat (2015), "On self selection of the corrupt into the public sector." *Economics Letters*, 127, 43–46.

- Bénabou, Roland and Jean Tirole (2006), “Incentives and Prosocial Behavior.” *The American Economic Review*, 96, 1652–1678.
- Bénabou, Roland and Jean Tirole (2011), “Identity, Morals, and Taboos: Beliefs as Assets.” *The Quarterly Journal of Economics*, 126, 805–855.
- Bernheim, B. Douglas (1994), “A theory of conformity.” *Journal of Political Economy*, 102, 841–877.
- Besley, Timothy and Maitreesh Ghatak (2005), “Competition and Incentives with Motivated Agents.” *The American Economic Review*, 95, 616–636.
- Besley, Timothy, Anders Jensen, and Torsten Persson (2014), “Norms, Enforcement, and Tax Evasion.” *Working Paper*.
- Besley, Timothy and John McLaren (1993), “Taxes and Bribery: The Role of Wage Incentives.” *The Economic Journal*, 103, 119–141.
- Bhattacharya, C B, Sankar Sen, and Daniel Korschun (2008), “Using Corporate Social Responsibility to Win the War for Talent.” *MIT Sloan Management Review*, 49, 37–44.
- Bidner, Chris and Patrick Francois (2013), “The Emergence of Political Accountability.” *The Quarterly Journal of Economics*, 128, 1397–1448.
- Bisin, Alberto and Thierry Verdier (2001), “The Economics of Cultural Transmission and the Dynamics of Preferences.” *Journal of Economic Theory*, 97, 298–319.
- Caselli, Francesco and Massimo Morelli (2004), “Bad politicians.” *Journal of Public Economics*, 88, 759–782.
- Cowley, Edd and Sarah Smith (2013), “Motivation and mission in the public sector: evidence from the World Values Survey.” *Theory and Decision*, 76, 241–263.
- Dal Bó, Ernesto, Frederico Finan, and Martín Rossi (2013), “Strengthening State Capabilities: The Role of Financial Incentives in the Call to Public Service.” *Quarterly Journal of Economics*, 128, 1169–1218.
- de Koning, Jaap and Paul de Hek (2013), “Wages in the Public Sector: a Comparison Between France, Germany, the Netherlands, Sweden and the United Kingdom.” *SEOR Report*.
- Delfgaauw, Josse and Robert Dur (2007), “Signaling and screening of workers’ motivation.” *Journal of Economic Behavior & Organization*, 62, 605–624.

- Delfgaauw, Josse and Robert Dur (2008), “Incentives and Workers’ Motivation in the Public Sector.” *The Economic Journal*, 118, 171–191.
- Deserranno, Erika (2015), “Financial incentives as signals: Experimental evidence from the recruitment of Health Workers.” *Working Paper*.
- Dewatripont, Mathias, Ian Jewitt, and Jean Tirole (1999), “The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies.” *Review of Economic Studies*, 66, 199–217.
- Di Tella, Rafael and Ernesto Schargrotsky (2003), “The Role of Wages and Auditing during a Crackdown on Corruption in the City of Buenos Aires.” *Journal of Law and Economics*, 46, 269–292.
- Dixit, Avinash (2002), “Incentive contracts for faith-based organizations to deliver social services.” In *Economic theory in a changing world: Policy modeling for growth* (Sajal Lahiri and Pradip Maiti, eds.), Oxford University Press.
- Dur, Robert and Robin Zoutenbier (2014), “Intrinsic Motivations of Public Sector Employees: Evidence for Germany.” *German Economic Review*, 16, 343–366.
- Ellingsen, Tore and Magnus Johannesson (2008), “Pride and Prejudice: The Human Side of Incentive Theory.” *The American Economic Review*, 98, 990–1008.
- Finan, Frederico, Benjamin Olken, and Rohini Pande (2015), “The Personnel Economics of the State.” *NBER Working Paper Series*, 1–48.
- Fischer, Paul and Steven Huddart (2008), “Optimal Contracting with Endogenous Social Norms.” *The American Economic Review*, 98, 1459–1475.
- Francois, Patrick (2000), “‘Public service motivation’ as an argument for government provision.” *Journal of Public Economics*, 78, 275–299.
- Francois, Patrick and Michael Vlassopoulos (2008), “Pro-social Motivation and the Delivery of Social Services.” *CESifo Economic Studies*, 54, 22–54.
- Granholm, Arne (2013), “När Professionerna Blev Byråkrater: Om statlig byråkrati under 200 år och hur traditionella professioners särställning minskat.” *Umeå Papers in Economic History*.
- Gregg, Paul, Paul Grout, Anita Ratcliffe, Sarah Smith, and Frank Windmeijer (2011), “How important is pro-social behaviour in the delivery of public services?” *Journal of Public Economics*, 95, 758–766.

- Gul, Faruk, Hugo Sonnenschein, and Robert Wilson (1986), “Foundations of dynamic monopoly and the coase conjecture.” *Journal of Economic Theory*, 39, 155–190.
- Handy, Femida and Eliakim Katz (1998), “The Wage Differential between Nonprofit Institutions and Corporations: Getting More by Paying Less?” *Journal of Comparative Economics*, 26, 246–261.
- Hanna, Rema and Shing-Yi Wang (2013), “Dishonesty and Selection into Public Service.” *NBER Working Paper Series*.
- Henderson, Rebecca and Eric Van den Steen (2015), “Why Do Firms Have “Purpose”? The Firm’s Role as a Carrier of Identity and Reputation.” *American Economic Review*, 105, 326–330.
- Huck, Steffen, Dorothea Kübler, and Jörgen Weibull (2012), “Social norms and economic incentives in firms.” *Journal of Economic Behavior & Organization*, 83, 173–185.
- Kosfeld, Michael and Ferdinand von Siemens (2011), “Competition, cooperation, and corporate culture.” *The RAND Journal of Economics*, 42, 23–43.
- Lazarsfeld, Paul and Robert Merton (1954), “Friendship as a Social Process: A Substantive and Methodological Analysis.” In *Freedom and Control in Modern Society* (Morroe Berger, Theodore Abel, and Charles Page, eds.), 18–66, Van Nostrand, New York.
- Macchiavello, Rocco (2008), “Public sector motivation and development failures.” *Journal of Development Economics*.
- Morris, Stephen and Hyun Song Shin (2000), “Rethinking Multiple Equilibria in Macroeconomic Modeling.” *NBER Macroeconomics Annual*, 15, 139–182.
- Murdock, Kevin (2002), “Intrinsic Motivation and Optimal Incentive Contracts.” *The RAND Journal of Economics*, 33, 650–671.
- Prendergast, Canice (2007), “The Motivation and Bias of Bureaucrats.” *The American Economic Review*, 97, 180–196.
- Prendergast, Canice (2008), “Intrinsic Motivation and Incentives.” *The American Economic Review*, 98, 201–205.
- Rose-Ackerman, Susan (1999), *Corruption and Government: Causes, Consequences, and Reform*. Cambridge University Press.

- Rothstein, Bo (2011), “Anti-corruption: the indirect ‘big bang’ approach.” *Review of International Political Economy*, 18, 228–250.
- Sliwka, Dirk (2007), “Trust as a Signal of a Social Norm and the Hidden Costs of Incentive Schemes.” *The American Economic Review*, 97, 999–1012.
- Sundell, Anders (2013), “Nepotism in the Swedish Central Public Administration 1790–1925.” *Working Paper*.
- The Global Journal (2015), *Top 500 NGOs*. Global Geneva.
- Tirole, Jean (1996), “A theory of collective reputations (with applications to the persistence of corruption and to firm quality).” *The Review of Economic Studies*, 63, 1–22.
- Transparency International (2014), *Corruption Perceptions Index 2014*.
- Treisman, Daniel (2000), “The causes of corruption: a cross-national study.” *Journal of Public Economics*, 76, 399–457.
- Van Rijckeghem, Caroline and Beatrice Weder (2001), “Bureaucratic corruption and the rate of temptation: do wages in the civil service affect corruption, and by how much?” *Journal of Development Economics*, 65, 307–331.
- Wilson, James (1989), *Bureaucracy: What Government Agencies Do and Why They Do It*. Basic Books.

## 7 (For Online Publication) Appendix A: Proofs

### 7.1 Proofs for Section 3

*Proof of Lemma 1:*

First, for non-motivated types,  $U_n(w_i) = w_i$  regardless of the place of employment. Since  $w_i(a_i = 1) = w^A$  and  $w_i(a_i = 0) = x_i$ , non-motivated workers have a best-response to set  $\hat{a}_i = 1$  iff  $x_i < w^A$ .

Second, for motivated types, by contradiction, assume that there exists an equilibrium with  $x_j < x_i$  and  $\hat{a}_j = 0$ ,  $\hat{a}_i = 1$ . This implies that  $w^A + v(C) > x_i$  and  $w^A + v(C) \leq x_j$ , a contradiction. ■

*Proof of Lemma 2:*

Since  $x^n = w^A$ , a high-motivation exists only if an equilibrium cutoff,  $x^m$ , exists in the range  $(w^A, \bar{x}]$ .

First, given  $w^A \in (\underline{x}, \bar{x})$ , if  $\bar{x} \leq w^A + v(C(\bar{x}))$  then a high-motivation equilibrium with  $x^n = w^A$  and  $x^m = \bar{x}$  exists. Moreover, it is the unique high-motivation equilibrium since  $U_m(x^m, C(x^m), a_i = 1) > U_m(x^m, a_i = 0)$  for  $x^m$  in  $[w^A, \bar{x}]$ . To see this, note that at  $x^m = w^A$ , where  $C = \lambda$ ,  $U_m(x^m, C(\lambda), a_i = 1) = w^A + v(\lambda) > U_m(x^m, a_i = 0) = w^A$ , and since  $v(C(x^m))$  is concave in  $x^m$ ,  $U_m(x^m, C(x^m), a_i = 1) > U_m(x^m, a_i = 0)$  over the whole interval  $[w^A, \bar{x}]$ .

Second, if  $\bar{x} > w^A + v(C(\bar{x}))$  then there exists a crossing of  $U_M(x^m, C(x^m), a_i = 1)$  and  $U_m(x^m, a_i = 0)$  for some  $x^m \in (w^A, \bar{x})$  by continuity, since  $U(x^m = w^A, C(\lambda), a_i = 1) > U_m(x^m = w^A, a_i = 0)$ . As above, uniqueness follows from the concavity of  $v(C(x^m))$  in  $x^m$ . ■

*Proof of Proposition 2:*

(i) By Lemma 2, a high-motivation exists for all values of  $w^A \in (\underline{x} - v(1), \bar{x})$ . Moreover, in the high-motivation equilibrium,  $\int_I a_i \rightarrow 0$  as  $w^{A+} \rightarrow (\underline{x} - v(1))$  and  $\int_I a_i \rightarrow 1$  as  $w^{A-} \rightarrow \bar{x}$ . Therefore, since both equilibrium cutoff values,  $x^n$  and  $x^m$ , are continuous in  $w^A$ , a high-motivation equilibrium with  $\int_I a_i = \nu$  exists from some value of  $w^A$ .

(ii.1) Given  $\nu < (1 - \lambda)(\underline{x} - v(0))$ , a market-clearing equilibrium exists where only non-motivated individuals select into institution A: For any  $w^A \in (\underline{x}, \underline{x} - v(0))$ , an equilibrium exists where  $x^m = \underline{x}$  since  $w^A + v(0) < \underline{x}$ . Moreover, over  $w^A \in (\underline{x}, \underline{x} - v(0))$ ,  $\int_I a_i$  is increasing continuously from 0 to  $-(1 - \lambda)(\underline{x} + v(0))$  in this low-motivation equilibrium, which implies a market-clearing equilibrium exists since  $\nu < (1 - \lambda)(\underline{x} - v(0))$ .

(ii.2) Follows from the proof of (ii.1). ■

*Proof of Proposition 3:*

First, we show that the optimal equilibrium cannot be a low-motivation equilibrium. By contradiction, assume that the optimal equilibrium,  $\{w^{A*}, C^*\}$ , is a low-motivation equilibrium and take  $a^* = \int_I \hat{a}_i$  for this equilibrium. By the proof of Proposition 2 (i), for any value of  $a^* \in (0, 1)$ , there exists a high-motivation equilibrium,  $\{w^A, C^h\}$  with  $\int_I a_i = a^*$ . Note that the gross social output is higher than  $\{w^{A*}, C^*\}$  in this high-motivation equilibrium. Moreover,  $w^A$  must be lower than  $w^*$ , due to the utility that motivated types receive from the higher level of reputation. Therefore, each interior low-motivation equilibrium is dominated by a high-motivation equilibrium, which implies that  $a^*$  cannot be interior. However,  $a^*$  cannot equal 0 by Assumption 1 and cannot equal 1 since  $\nu < 1$ , which implies that  $a^* = 1$  is dominated by the market-clearing high-motivation equilibrium.

Second, to partially characterize the optimal equilibrium, we can utilize the comparative statics of  $C$  in the unique high-motivation equilibrium with respect to  $w^A$ . Note that result follows if  $\partial C(x^m, x^n) / \partial w^A \leq 0$ , since the market-clearing high-motivation equilibrium will have higher net social output than an equilibrium with

a higher wage and lower average motivation.

Starting from  $w^{A^h}$ , consider a wage increase of  $\Delta w$  to  $w^{A''}$ . Note that  $\Delta x^n = \Delta w$ . Assume, by contradiction, that in the unique high-motivation equilibrium at  $w^{A''}$ ,  $C(x^{m''}, x^{n''})$  is equal to  $C^{h'}$ . This implies that  $\Delta x^m = \Delta x^n = \Delta w$ , since the reputation-payoffs are the same after the wage increase, and utility is linear with respect to wage. However, if  $\Delta x^m = \Delta x^n$ , then  $C(x^{m''}, x^{n''}) < C^{h'}$ , since  $C^{h'} > \lambda$ . Therefore, at  $x^{m''} = x^{m'} + \Delta w$ ,  $x^{m''} > w^{A''} + v(C(x^{m''}, x^{n''}))$ , which implies that, given  $w^{A''}$ , the corresponding high-motivation equilibrium has  $C'' < C^{h'}$ . This shows that at any high-motivation equilibrium with  $w^A > w^{A^h}$ ,  $C < C^{h'}$ , which proves the result. ■

## 7.2 Proofs for Section 4.1

*Proof of Lemma 3:*

To extend Lemma 1 to the dynamic case, it suffices to show that, by type,  $O_i^t$  is monotonically decreasing in  $x^i$ , since this will preserve the single-crossing of  $U^t(m_i, x_i, a_i^t = 1)$  and  $U^t(m_i, x_i, a_i^t = 0)$ . Note that for  $i, j$  of the same motivation-type with  $x_j < x_i$ , the relative utility of employment in  $A$  is higher for  $j$  in *each* period, since the relative period utility is equal to  $u_t(w_i^A, C_{t-1}, m_i, a_i = 1) - x_i$ . Therefore,  $O_i^t$  is strictly decreasing in  $x_i$ , which implies the single-crossing condition. ■

*Proof of Proposition 4:*

The proof of Proposition 4 follows from an equivalence of the conditions for a steady-state equilibrium and the equilibrium conditions for a static equilibrium, given the constant collective reputation in the steady-state. Specifically, in a steady-state, the option value of employment in  $A$  for the cutoff types is equal to zero: By contradiction, assume  $O_i^t$  is strictly positive for  $x_i = \bar{x}^m$ ,  $m_i = 1$ . Since  $C_t, w^A$  are constant for all  $t$ ,  $O_i^t$  is strictly positive only if  $\bar{x}^m < \bar{w}^A + v(\bar{C})$ . However, this implies that  $\bar{x}^m < \bar{w}^A + v(\bar{C}) + O_i^t(x^m)$ , which is a contradiction.

Given that the cutoff types are indifferent between  $\hat{a}_i = 0, 1$  in each period,  $\{\bar{w}^A, \bar{x}^m, \bar{x}^n, \bar{C}\}$  define a steady-state equilibrium if and only if the following conditions are satisfied:

$$\begin{aligned}\bar{x}^m &= \bar{w}^A + v(\bar{C}), \\ \bar{x}^n &= \bar{w}^A, \\ \bar{C} &= \frac{\lambda(\bar{x}^m - \underline{x})}{(1 - \lambda)(\bar{x}^n - \underline{x}) + \lambda(\bar{x}^m - \underline{x})},\end{aligned}$$

which are equivalent to the conditions for a static equilibrium. ■

*Proof of Corollary 1:*

Since the period payoffs in a steady-state equilibrium are equivalent to the payoffs

of the corresponding static equilibrium,  $\Pi^A = \sum(1 - \tau)^{t-1}\bar{\pi}^A$  is maximized at the steady-state that maximizes  $\bar{\pi}^A$ . ■

*Proof of Lemma 4:*

The proof of uniqueness follows from the fact that workers maximize their objective using the following simple decision rule:

$$\hat{a}_{i,t} = 1 \text{ iff } w_t^A + m_i v(C_{t-1}) \geq x_i,$$

which implies a unique value for both  $x^m$ ,  $x^n$ . Additionally, this equation shows that  $x^m > x^n = w^A$  iff  $v(C_{t-1}) > 0$ , which implies the second result. ■

*Proof of Lemma 5:*

First, we give an expression for  $C_t$  as a function of the cutoff types:

$$C_t = \frac{\int_I \hat{a}_{i,t} m_i}{\int_I \hat{a}_{i,t}} = \frac{\lambda(x_t^m - \underline{x})}{(1 - \lambda)(x_t^n - \underline{x}) + \lambda(x_t^m - \underline{x})}$$

Due to the quasi-linearity of both type's utility with respect to the wage, for interior values  $\partial x_t^m(w_t^A, C_{t-1})/\partial w_t^A = \partial x_t^n(w_t^A, C_{t-1})/\partial w_t^A = 1$ , which implies that:

$$\partial C^t / \partial w_t^A = \frac{\lambda((1 - \lambda)(x_t^n(w_t^A) - \underline{x}) + \lambda(x_t^m(w_t^A, C_{t-1}) - \underline{x})) - \lambda(x_t^m(w_t^A) - \underline{x})}{((1 - \lambda)(x_t^n(w_t^A) - \underline{x}) + \lambda(x_t^m(w_t^A, C_{t-1}) - \underline{x}))^2}$$

This expression is negative iff:

$$(1 - \lambda)x_t^n(w_t^A) + \lambda x_t^m(w_t^A, C_{t-1}) < x_t^m(w_t^A),$$

which is true iff  $x_t^m(w_t^A, C_{t-1}) > x_t^n(w_t^A)$ .

Next, note that the relationship between  $x_t^m(w_t^A, C_{t-1})$  and  $x_t^n(w_t^A)$  depends only on the sign of  $v(C_{t-1})$ , since motivated types' utility is separable with regard to the wage and reputation. In particular:

$$x_t^m(w_t^A, C_{t-1}) \leq x_t^n(w_t^A, C_{t-1}) \text{ iff } v(C_{t-1}) \leq 0.$$

Lastly, note that the same relationship holds when one of the two cutoffs is non-interior, and when both are non-interior,  $\partial C^t / \partial w_t^A = 0$ . ■

*Proof of Lemma 6:*

The statement of Lemma 6 is equivalent to:

1. For  $\{w_t^A\}$  s.t.  $w_{t+1}^A \leq w_t^A$ ,  $C_t < C^*$  for all  $t$ .
2. For  $\{w_t^A\}$  s.t.  $w_{t+1}^A \geq w_t^A$ ,  $C_t < C^*$  for all  $t$ .

(1) follows directly from Lemma 5: given  $v(C_0) < 0$ ,  $\partial C_t(w_t^A)/\partial w_t^A \leq 0$ , which implies  $C_t \leq C_0$  for all  $t$ .

(2) By contradiction, assume there exists  $\{w_t^A\}$  such that  $w_{t+1}^A \geq w_t^A$ , and  $C_t \geq C^*$  for some  $t$ . Take  $t$  equal to  $\min_t\{t|C_t \geq C^*\}$ . It follows that  $C_{t-1} < C^*$ , and therefore  $\partial C_t(w_t^A)/\partial w_t^A > 0$ . By Lemma 5, this implies that  $v(C_{t-1}) \leq 0$ . However, by Lemma 4,  $v(C_{t-1}) \leq 0$  implies that  $C_t \leq \lambda$ , a contradiction since  $\lambda < C^*$ . ■

*Proof of Proposition 5:*

Given  $\{w^{A^*}, C^*\} = \{w^{A^h}, C^{h'}\}$  and  $\{w^{A^h}, C^{h'}\}$  unique, existence of a transition given  $v(\lambda) > 0$  follows from the example provided in the main text.

If this is not the case, then the following wage path transitions to  $\{w^{A^*}, C^*\}$ :

1.  $w_1^A$  solves  $w_1^A + v(C_0) = \bar{x}$ ; that is,  $w_1^A$  is set high enough that  $\hat{a}_{i,1} = 1$  for all  $i$ .
2.  $w_2^A$  solves  $C_2 = C^*$ .
3.  $w_t^A$  for  $t > 2$  equals  $w^{A^*}$ .

Note that there exists  $w_2^A$  such that  $C_2 = C^*$ , since given  $v(C_1 = \lambda) > 0$ ,  $C_2(w_2^A)$  is a continuous function with a range of  $[\lambda, 1]$ . ■

**In the following proofs, for purposes of exposition it will occasionally be useful for us to explicitly refer to expectation over wages, reputation and demand for jobs in institution  $A$ , which we denote  $\{\tilde{w}_t^A\}$ ,  $\{\tilde{C}_t\}$  and  $\{\tilde{q}_t\}$ .**

*Proof of Proposition 6:*

We show the result by construction. Take  $\{w_t^A\}$  such that:

1.  $w_t^A + v(C_0) = \bar{x}$  for  $t < t'$ , where  $t' = \min\{t|v(C_{t'-1}) > 0\}$ .
2.  $w^{A'}$  where  $w^{A'}$  solves  $C_t = C^*$  given  $\tilde{q}_{t'+1} = 1$  for  $t = t'$ .
3.  $w^{A^*}$  for  $t > t'$ .

To show that (1) leads to  $C_{t'-1}$  such that  $v(C_{t'-1}) > 0$ , note that  $\hat{a}_{i,1} = 1$  for all workers independent of expectations of  $q_t$ . Therefore, given  $w_t^A = \bar{x} - v(C_0)$  and  $\int_I a_{i,t} m_i / \int_I a_{i,t} < \lambda$ ,  $C_t$  is strictly increasing since a measure  $\delta\nu$  of workers will join institution  $A$  from the set of workers with  $a_{i,t} = 0$ , who have an average motivation greater than  $\lambda$ . This implies that, given  $w_t^A = \bar{x} - v(C_0)$ , the sequence  $\{C_t\}$  converges to  $\lambda$  as  $t \rightarrow \infty$ , and  $\{C_t\}$  is unique (independent of expectations). This shows that, since  $v(\lambda) > 0$ , there exists  $t'$  such that  $v(C_{t'-1}) > 0$ , and that  $t'$  is unique.

Next we show that (2) gives an equilibrium that transitions to  $\{w^{A^*}, C^*\}$ . Note that the distance between  $x_t^m$  and  $x_t^n$  is fixed by  $v(C_{t-1})$  when  $q_{t+1} = 1$ . This implies that, given  $q_{t+1} = 1$  and  $v(C_{t-1}) > 0$ ,  $C_t$  is decreasing function of  $w_t^A$  with a range of  $[\lambda, 1]$ . Therefore, at  $t'$  there exists a  $w_{t'}^A$  such that given  $q_{t'+1} = 1$ ,  $C_{t'} = C^*$ . Lastly, note that an equilibrium exists with  $\tilde{q}_{t'+1} = 1$  since, at  $\{w^{A^*}, C^*\}$ ,  $\int_I a_i = \nu$ . ■

*Proof of Proposition 7:*

Here we show that, given the wage path detailed in the proof of Proposition 6, any equilibrium transitions to some  $C^* + \epsilon$ , where  $\epsilon \geq 0$ . Note that we have already shown that  $\{w_t^A, C_{t-1}\}_0^{t'-1}$  is unique. However, multiple equilibrium may exist for

$\{w_t^A, C_t\}_{t'}^\infty$ . Take the set of reputations in the transition equilibrium detailed above to be  $\{w_t^{A''}, C_{t-1}''\}$ ; we will show, by contradiction, that for any other equilibrium  $\{w_t^{A''}, C_{t-1}\}$ ,  $C_t'' \leq C_t$ , which proves the result.

Assume there exists an equilibrium  $\{w_t^{A''}, C_{t-1}\}$  where  $C_t'' > C_t$  for some  $t \geq t'$ . First, note that  $\{w_t^{A''}, C_{t-1}''\}$  is unique given expectations that  $\tilde{q}_t = 1$ : if  $q_t = 1$  in all periods, then workers will simply select employment that maximizes their period payoffs, which will result in an equilibrium of  $\{w_t^{A''}, C_{t-1}''\}_{t'}^\infty$ . Therefore, other equilibria will only occur under expectations that  $\tilde{q}_t < 1$  for some set of periods. However, given  $w_t^{A''}$ , institution  $A$  will only be over-demanded,  $\tilde{q}_t < 1$ , if  $C_{t-1}'' < C_{t-1}$ .

Take  $\underline{t}$  equal to the minimum value of  $t$  where  $\tilde{q}_t < 1$ , and  $\bar{t}$  equal to the minimum value of  $t$  where  $C_t'' > C_t$ . It must be the case that  $\bar{t} > \underline{t}$ , since  $C_t'' = C_t$  for  $t < \underline{t}$ , and since  $\tilde{q}_t < 1$  only if  $C_t'' < C_t$ . Moreover, given  $C_t'' < C_t$ ,  $C_{t+1}'' < C_{t+1}$  since  $x_{t+1}^{m''} < x_{t+1}^m$  and  $x_t^n$  is unchanged since, given  $w_t^{A''}$  is non-increasing, the option value of  $a_i = 1$  is zero for the non-motivated cutoff type. This in turn implies, by induction, that  $C_t'' < C_t$  for all  $t \geq \underline{t}$ . Therefore, given  $\{w_t^{A''}\}$ , all equilibria correspond to a sequence of  $\{C_{t-1}\}$  that are bounded below (weakly) by  $\{C_{t-1}''\}$ , which contradicts the existence of an equilibrium  $\{w_t^{A''}, C_{t-1}\}$  where  $C_t'' > C_t$ . ■

*Proof of Proposition 8:*

Given that the proof of Proposition 7 establishes transition once a point with  $v(C_{t-1}) > 0$  is reached, it suffices to show that setting a wage of  $\underline{x} + v(0) + \epsilon$  will cause a transition to a point such that  $v(C_{t-1}) > 0$  in all equilibria. Note that if  $w^A = \underline{x} + v(0) + \epsilon$ , then there is a unique static (steady-state) equilibrium, since  $U_m(x^m, a_i = 1)$  cannot intersect  $U_m(x^m, a_i = 0)$  from below. Moreover, since it is unique, it is a high-motivation equilibrium by Lemma 2, which implies that  $U_m(x^m, a_i = 1) > U_m(x^m, a_i = 0)$  over the domain  $[\underline{x}, \underline{x} + v(0) + \epsilon]$ .

Next, take  $\{C_t', x_t^{m'}, x_t^{n'}\}$  to be the set of reputation and cutoff abilities that result from myopic behavior, where agents choose  $\hat{a}_i$  to maximize their period payoffs given  $w_t^A = \underline{x} + v(0) + \epsilon$  and  $C_0$ . Given the constant wage,  $x_t^{n'}$  is constant and equal to  $\underline{x} + v(0) + \epsilon$ . However, given  $C_t < C_t^h$  where  $C_t^h$  is the unique steady-state equilibrium,  $U_{m,t}(C(x_{t-1}^m), a_i = 1) > U_{m,t}(C(x_{t-1}^m), a_i = 0)$  for all  $t > 2$ , which implies that  $x_t^{m'} > x_{t-1}^{m'}$ . Moreover, the same logic implies that  $x_t^{m'}$  converges to the cutoff value in the unique, high-motivation static equilibrium. Therefore, given  $\underline{x} + v(0) + \epsilon$ ,  $\{C_t'\}$  converges to  $C_t^h$ , which implies that  $v(C_{t-1}') > 0$  for some  $t$ .

We now show that, given  $w_t^A = \underline{x} + v(0) + \epsilon$ , all equilibrium sequences of  $\{C_t\}$  are bounded below by  $\{C_t'\}$ . First, note that since wages are non-increasing,  $O_i^t = 0$  for  $x_t^n$ , which implies that  $x_t^n = x_t^{n'}$  in all equilibria. We can then prove the result by induction:  $C_1 \geq C_1'$  since  $x_1^m = x_1^{m'}$  unless the unless  $A$  is over-demanded in period 2, in which case  $x_1^m \geq x_1^{m'}$ . Moreover, by the same argument, given  $C_{t-1} \geq C_{t-1}'$ , it must

be true that  $C_t \geq C'_t$ .

This proves that, given  $w_t^A = \underline{x} + v(0) + \epsilon$ ,  $\{C_t\}$  is bounded below by  $\{C'_t\}$  in equilibrium, which implies that for all equilibria, there exists  $t$  such that  $v(C'_{t-1}) > 0$ . ■

### 7.3 Proofs for Section 5

*Proof of Proposition 1':*

*High-motivation:* Given  $\nu < \lambda(\underline{x} + v(1))$ , a market-clearing equilibrium exists where only motivated individuals select into institution  $A$ . For  $w^A \in (\underline{x} - v(1), \underline{x})$ ,  $x^n = \underline{x}$  in all equilibria since  $w^A < \underline{x}$ . However, a high-motivation equilibrium exists with  $x^m = w^A + v(1)$ , where  $C = 1$  since all non-motivated workers select into  $B$ . Moreover, over  $w^A \in (\underline{x} - v(1), \underline{x})$ ,  $\int_I a_i$  is increasing continuously from 0 to  $\lambda(\underline{x} + v(1))$  in this high-motivation equilibrium, which implies a market-clearing equilibrium exists since  $\nu < \lambda(\underline{x} + v(1))$ .

*Low-motivation:* Note that if  $v(\lambda) = 0$  then a crossing of  $U_m(w^A, C(x^m), a_i = 1)$  and  $U_m(w^A, C(x^m), a_i = 0)$  exists at  $x^m = x^n = w^A$  for  $w^A \in (\underline{x}, \bar{x})$ . Therefore, a low-motivation market-clearing equilibrium exists at  $w^A = \underline{x} + \nu$ . ■

*Proof of Proposition 4':*

Non-existence given  $v(\lambda) = 0$  follows as a corollary to the proof of Lemma 4. By contradiction, assume  $v(\lambda) = 0$ ,  $v(C_0) < 0$  and  $\{w_t^A\}$  such that a transition to  $\{w_t^{A*}, C^*\}$  is an equilibrium. Since  $v(C_0) < 0$ , it follows that  $C_1 < \lambda$ . Therefore, for a transition to exist, it must be true that  $C_t \leq \lambda$  and  $C_{t+1} > \lambda$  for some  $t$ .

However, if  $C_t = \lambda$ , then  $C_{t+1} = \lambda$  since  $x_{t+1}^m(w_{t+1}^A, C_t) = x_{t+1}^n(w_{t+1}^A)$  when  $v(C_t) = 0$ . If  $C_t < \lambda$ , then  $v(C_t) < 0$  and by the proof of Lemma 4,  $C_{t+1} < \lambda$ , which contradicts  $C_{t+1} > \lambda$ . ■

*Proof of Proposition 5':*

Note that for a transition to occur along  $\{w^A\}$ , it must be true that  $C_{t-1} \leq \lambda$  and  $C_t > \lambda$  for some  $t$ . Since  $v(C_{t-1}) \leq 0$ , it follows that, since  $C_t > \lambda$ ,  $O_i^t(m_i = 1, x'_i) > O_i^t(m_i = 0, x'_i)$ ; that is, holding private-sector ability constant, the option value of public sector employment must be higher for a motivated worker than a non-motivated worker in period  $t$ . Additionally, for the option value of the motivated worker to be higher, it must be true that workers believe that the public sector will be over-demanded for some set  $\{t+1, \dots, t+n\}$  and  $C_{t'} > \lambda$  for some  $t' \in \{t+1, \dots, t+n\}$ .

However, given  $\{w^A\}$ , take the set of beliefs  $\{\{\tilde{C}_{t+1}, \dots, \tilde{C}_{t+n}\}\}$  such that  $\tilde{C}_{t'} \leq \lambda$  for all  $t' \in \{t+1, \dots, t+n\}$ . With any beliefs in this set,  $O_i^{t'}(m_i = 1, x'_i) < O_i^{t'}(m_i = 0, x'_i)$ , implying that  $C_{t'} \leq \lambda$  for  $t' \in \{t+1, \dots, t+n\}$ . Therefore, under these beliefs,  $O_i^t(m_i = 1, x'_i) < O_i^t(m_i = 0, x'_i)$ , implying that some beliefs  $\{\tilde{C}_{t+1}, \dots, \tilde{C}_{t+n}\}$  in this set

are “self-filling,” in the sense that they constitute an equilibrium where  $C_t \leq \lambda$ . This shows that for any  $\{w^A\}$  that admits a transition in equilibrium, there is an alternative set of beliefs such that there is no transition. ■

*Proof of Proposition 9:*

For (i) note that if motivated workers value the ability-contingent reputation, then the model in this section reduces to the model presented in the general analysis – therefore the result follows from Propositions 7 and 5’.

For (ii), a transition can be achieved with a wage path,  $\{w_t^{A,p}\}$ , where:

$$w_t^{A,p} \begin{cases} = 0 & \text{if } \lambda_p \leq \bar{\lambda} \\ = \bar{x}^p + v(C_{t-1}) & \text{if } \lambda_p > \bar{\lambda}, \end{cases}$$

until  $t'$  such that  $v(C_{t'-1}) > 0$ , and  $w_t^{A,p}$  is market-clearing for  $t \geq t'$ . Note that, in contrast to the main analysis with  $v(\lambda) = 0$ ,  $t'$  exists since an ability level exists with  $\lambda^p > \bar{\lambda}$  by the assumption that  $\lambda^i \neq \lambda^j$  for some  $i, j \in \{1, \dots, p\}$ , and since  $C_t \rightarrow \sum^{p'} \lambda^p / |p'| > \bar{\lambda}$  as  $t \rightarrow \infty$ , where  $p' \equiv \{p | \lambda_p > \bar{\lambda}\}$ . That is, a reputation greater than  $\bar{\lambda}$  is enabled by the fact that institution can achieve a high-motivation reputation by selectively raising wages in ability-level that have a higher proportion of motivated types. Give  $v(C_{t'-1}) > 0$ , a transition to  $\{w^{A*}, C^*\}$  is achieved by the same argument as in the proof of Proposition 7. ■

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