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Abstract

College Admissions with Entrance Exams: Centralized versus Decentralized

by Isa E. Hafalir, Rustamdjan Hakimov, Dorothea Kübler and Morimitsu Kurino *

We theoretically and experimentally study a college admissions problem in which colleges accept students by ranking students' efforts in entrance exams. Students hold private information regarding their ability level that affects the cost of their efforts. We assume that student preferences are homogeneous over colleges. By modeling college admissions as contests, we solve and compare the equilibria of "centralized college admissions" (CCA) in which students apply to all colleges, and "decentralized college admissions" (DCA) in which students can only apply to one college. We show that lower ability students prefer DCA whereas higher ability students prefer CCA. The main qualitative predictions of the theory are supported by the experimental data, yet we find a number of behavioral differences between the mechanisms that render DCA less attractive than CCA compared to the equilibrium benchmark.

Keywords: College admissions, incomplete information, student welfare, contests, all-pay auctions, experiment

JEL classification: C78; D47; D78; I21

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1 Introduction

Throughout the world and every year, millions of prospective university students apply for admission to colleges or universities during their last year of high school. Admission mechanisms vary from country to country, yet in most countries there are government agencies or independent organizations that offer *standardized admission exams* to aid the *college admission* process. Students invest a lot of time and effort to do well in these admission exams, and they are heterogeneous in terms of their ability to do so.

In some countries, the application and admission process is centralized. For instance, in Turkey university assignment is solely determined by a national examination called YGS/LYS. After learning their scores, students can apply to a number of colleges. Applications are almost costless as all students need only to submit their rank-order of colleges to the central authority.¹ On the other hand, Japan has a centralized “National Center test,” too, but all public universities including most prestigious universities require the candidate to take another, institution-specific secondary exam which takes place on the same day. This effectively prevents the students from applying to more than one public university.² The admissions mechanism in Japan is decentralized, in the sense that colleges decide on their admissions independent of each other. In the United States, students take both centralized exams like the Scholastic Aptitude Test (SAT), and also complete college-specific requirements such as college admission essays. Students can apply to more than one college, but since the application process is costly, students typically send only a few applications (the majority being between two to six applications, see Chade, Lewis, and Smith, 2014). Hence, the United States college admissions mechanism falls in between the two extreme cases.

In this paper, we compare the institutional effects of different college admission mechanisms on the equilibrium efforts of students and student welfare. To do this, we model college admissions with admission exams as contests (or all-pay auctions) in which the cost of effort represents the payment made by the students. We focus on two extreme cases: in the centralized model (as in the Turkish mechanism) students can freely apply to all colleges, whereas in the decentralized model (as in the Japanese mechanism for public colleges) students can only apply to one college. For simplicity, in our main model we consider two colleges that differ in quality and assume that

¹Greece, China, South Korea, and Taiwan have similar national exams that are the main criterion for the centralized mechanism of college admissions. In Hungary, the centralized admission mechanism is based on a score that combines grades from school with an entrance exam (Biro, 2012).

²There are actually two stages where the structure of each stage is as explained in Section 4. The difference between the stages is that the capacities in the first stage are much greater than those in the second stage. Those who do not get admission to any college spend one year preparing for the next year’s exam. Moreover, the Japanese high school admissions authorities have adopted similar mechanisms in local districts. Although the mechanism adopted varies across prefectures and is changing year by year, its basic structure is that each student chooses one among a specified set of public schools and then takes an entrance exam at his or her chosen school. The exams are held on the same day. Finally, institution-specific exams that prevent students from applying to all colleges have also been used and debated in the United Kingdom, notably between the University of Cambridge and the University of Oxford. We thank Ken Binmore for pointing this out.

students have homogeneous preferences for attending these colleges.³

More specifically, each of the n students gets a utility of v_1 by attending college 1 (which can accommodate q_1 students) and gets a utility of v_2 by attending college 2 (which can accommodate q_2 students). We suppose $0 < v_1 < v_2$, and hence college 2 is the better and college 1 is worse of the two colleges. Students' utility from not being assigned to any college is normalized to 0. Following the majority of the literature on contests with incomplete information, we suppose that an ability level in the interval $[0, 1]$, is drawn i.i.d. from the common distribution function, and the cost of exerting an effort e for a student with ability level a is given by $\frac{e}{a}$. Thus, given an effort level, the higher the ability the lower the cost of exerting the effort.

In the centralized college admissions problem (CCA), all students rank college 2 over college 1. Hence, the students with the highest q_2 efforts get into college 2, students with the next highest q_1 efforts get into college 1, and students with the lowest $n - q_1 - q_2$ efforts are not assigned to any college. In the decentralized college admissions problem (DCA), students need to simultaneously choose which college to apply to and how much effort to exert. Then, for each college $i \in \{1, 2\}$, students with the highest q_i efforts *among the applicants to college i* get into college i .

It turns out that the equilibrium of CCA can be solved by standard techniques, such as in Moldovanu, Sela, and Shi (2012). In this monotone equilibrium, higher ability students exert higher efforts, and therefore the students with the highest q_2 ability levels get admitted to the good college (college 2), and students with ability rankings between $q_2 + 1$ and $q_1 + q_2$ get admitted to the bad college (college 1) (Proposition 1).

Finding the equilibrium of DCA is not straightforward. It turns out that in equilibrium, there is a cutoff ability level that we denote by c . All higher ability students (with abilities in $(c, 1]$) apply to the good college, whereas lower ability students (with ability levels in $[0, c]$) use a *mixed strategy* when choosing between the good and the bad college. Students' effort functions are continuous and monotone in ability levels (Theorem 1). Our paper therefore contributes to the all-pay contests literature. To the best of our knowledge, ours is the first paper to model and solve "competing contests" where the players have private information regarding their abilities and sort themselves into different contests.

After solving for the equilibrium of CCA and DCA, we compare the equilibria in terms of students' interim expected utilities. We show that students with lower abilities prefer DCA to CCA when the number of seats is smaller than the number of students (Proposition 2). The main intuition for this result is that students with very low abilities have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero due to the fewer number of applications than the capacity. Moreover, we show that students with higher abilities prefer CCA to DCA (Proposition 3).⁴ The main intuition for this result is that

³In Section 6, we discuss the case with three or more colleges.

⁴More specifically we obtain a single crossing condition: if a student who applies to college 2 in the decentralized mechanism prefers the centralized mechanism to the decentralized mechanism, then all higher ability students also

high-ability students (i) can only get a seat in the good school in DCA, whereas they can get seats in both the good and the bad school in CCA, and (ii) their equilibrium probability of getting a seat in the good school is the same across the two mechanisms.

We test the theory with the help of lab experiments. We implement five markets for the college admissions game that are designed to capture different levels of competition (in terms of the supply of seats, the demand ratio, and the quality difference between the two colleges). We compare the two college admission mechanisms and find that in most (but not all) markets, the comparisons of the students' ex-ante expected utilities, their effort levels, and the students' preferences regarding the two college admission mechanisms are well organized by the theory. However, the experimental subjects exert a higher effort than predicted. The overexertion of effort is particularly pronounced in DCA, which makes it relatively less attractive for the applicants compared to CCA.

The rest of the paper is organized as follows. The introduction (Section 1) ends with a discussion of the related literature. Section 2 introduces the model and preliminary notation. In sections 3 and 4 we solve the model for the Bayesian Nash equilibria of the centralized and decentralized college admission mechanisms, respectively. Section 5 offers comparisons of the equilibria of the two mechanisms. Section 6 discusses the case of three or more colleges. Section 7 presents our experimental results. Finally, section 8 concludes. Omitted proofs are given in the Appendix.

1.1 Related literature

College admissions have been studied extensively in the economics literature. Following the seminal paper by Gale and Shapley (1962), the theory literature on two-sided matching mainly considers centralized college admissions and investigates stability, incentives, and the efficiency properties of various mechanisms, notably the deferred-acceptance and the top trading cycles algorithms. The student placement and school choice literature is motivated by the centralized mechanisms of public school admissions, rather than by the decentralized college admissions mechanism in the US. This literature was pioneered by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003). We refer the reader to Sönmez and Ünver (2011) for a recent comprehensive survey regarding centralized college admission models in the two-sided matching literature. Recent work regarding centralized college admissions with entrance exams include Abizada and Chen (2011) and Tung (2009). Abizada and Chen (2011) model the entrance (eligibility) criterion in college admissions problems and extend models of Perach, Polak, and Rothblum (2007) and Perach and Rothblum (2010) by allowing the students to have the same scores from the central exam. On the other hand, by allowing students to submit their preferences after they receive the test results, Tung (2009) adjusts multi-category serial dictatorship (MSD) analyzed by Balinski and Sönmez (1999) in order to make students better off.

One crucial difference between the modelling in our paper and the literature should be emphasized.

have the same preference ranking.

phasized: In our paper student preferences affect college rankings over students through contests among students, while student preferences and college rankings are typically independent in the two-sided matching models and school-choice models.

The analysis of decentralized college admissions in the literature is more recent. Chade, Lewis, and Smith (2014) consider a model where two colleges receive noisy signals about the caliber of applicants. Students need to decide which colleges to apply to and application is costly. The two colleges choose admissions standards that act like market-clearing prices. The authors show that in equilibrium, college-student sorting may fail, and they also analyze the effects of affirmative action policies. In our model, the colleges are not strategic players as in Chade, Lewis, and Smith (2014). Another important difference is that in our model the students do not only have to decide which colleges to apply to, but also how much effort to exert in order to do well in the entrance exams. Che and Koh (2013) study a model in which two colleges make admission decisions subject to aggregate uncertainty about student preferences and linear costs for any enrollment exceeding the capacity. They find that colleges' admission decisions become a tool for strategic yield management, and in equilibrium, colleges try to reduce their enrollment uncertainty by strategically targeting students. In their model, as in Chade, Lewis, and Smith (2014), students' exam scores are costlessly obtained and given exogenously. Avery and Levin (2010), on the other hand, analyze a model of early admission at selective colleges where early admission programs give students an opportunity to signal their enthusiasm to the college they would like to attend.

In another related paper, Hickman (2009) also models college admissions as a Bayesian game where heterogeneous students compete for seats at colleges. He presents a model in which there is an allocation mechanism mapping each student's score into a seat at a college. Hickman (2009) is mostly interested in the effects of affirmative action policies, and the solution concept used is "approximate equilibrium" in which the number of students is assumed to be large so that students approximately know their rankings within the realized sample of private costs.⁵ In our paper, we do not require the number of students to be large. In another recent paper by Salgado-Torres (2013), students and colleges participate in a decentralized matching mechanism called Costly Signaling Mechanism (CSM) in which students first choose a costly observable score to signal their abilities, then each college makes an offer to a student, and finally each student chooses one of the available offers. Salgado-Torres (2013) characterizes a symmetric equilibrium of CSM which is proven to be assertive, and also performs some comparative statics analysis. CSM is decentralized just like the decentralized college admissions model developed in this paper. However, CSM cannot be used to model college admission mechanisms (such as the ones used in Japan) that require students to apply to *only one* college.

Our paper is also related to the all-pay auction and contests literature. Notably, Baye,

⁵In a related paper, Morgan, Sisak, and Vardy (2012) study competition for promotion in a *continuum economy*. They show that a more meritocratic profession always succeeds in attracting the highest ability types, whereas a profession with superior promotion benefits attracts high types only under some assumptions.

Kovenock, and de Vries (1996) and Siegel (2009) solve for all-pay auctions and contests with complete information. We refer the reader to the survey by Konrad (2009) about the vast literature on contests. Related to our decentralized mechanism, Amegashie and Wu (2006) and Konrad and Kovenock (2012) both model “competing contests” in a *complete information* setting. Amegashie and Wu (2006) study a model where one contest has a higher prize than the other. They show that sorting may fail in the sense that the top contestant may choose to participate in the contest with a lower prize. In contrast, Konrad and Kovenock (2012) study all-pay contests that are run simultaneously with multiple identical prizes. They characterize a set of pure strategy equilibria, and a symmetric equilibrium that involves mixed strategies. In our decentralized college admissions model, the corresponding contest model is also a model of competing contests. The main difference in our model is that we consider incomplete information as students do not know each other’s ability levels.

A series of papers by Moldovanu and Sela (and Shi) studies contests with incomplete information, but they do not consider competing contests in which the participation in contests is endogenously determined. In Moldovanu and Sela (2001), the contest designer’s objective is to maximize expected effort. They show that when cost functions are linear or concave in effort, it is optimal to allocate the entire prize sum to a single first prize. Moldovanu and Sela (2006) compare the performance of dynamic sub-contests whose winners compete against each other with static contests. They show that with linear costs of effort, the expected total effort is maximized with a static contest, whereas the highest expected effort can be higher with contests with two divisions. Moldovanu, Sela, and Shi (2012) study optimal contest design where both awards and punishments can be used. Under some conditions, they show that punishing the bottom is more effective than rewarding the top.

This paper also contributes to a large experimental literature on contests and all-pay auctions, summarized in a recent survey article by Dechenaux, Kovenock, and Sheremeta (2012). Our setup in the centralized mechanism with heterogeneous agents, two non-identical prizes, and incomplete information is closely related to a number of existing studies by Barut, Kovenock, and Noussair (2002), Noussair and Silver (2006), and Müller and Schotter (2010). These studies observe that agents overbid on average compared to the Nash prediction. Moreover, they find an interesting “bifurcation,” a term introduced by Müller and Schotter (2010), in that low types underbid and high types overbid. Regarding the optimal prize structure, it turns out that if players are heterogeneous, multiple prizes can be optimal to avoid the discouragement of weak players, see Müller and Schotter (2010). Higher effort with multiple prizes than with a single prize was also found in a setting with homogeneous players by Harbring and Irlenbusch (2003).

We are not aware of any previous experimental work related to our decentralized admissions mechanism where agents simultaneously have to choose an effort level and decide whether to compete for the high or the low prize.

The paper also belongs to the experimental literature on two-sided matching mechanisms and

school choice starting with Kagel and Roth (2000) and Chen and Sönmez (2006).⁶ These studies as well as many follow-up papers in this strand of the literature focus on the rank-order lists submitted by students in the preference-revelation games, but not on effort choice. Thus, the rankings of students by the schools are exogenously given in these studies unlike in our setup where the colleges' rankings are endogenous.

2 The Model

The college admissions problem with entrance exams, or simply the problem, is denoted by $(S, \mathcal{C}, (q_1, q_2), (v_1, v_2), F)$. There are 2 colleges – college 1 and college 2. We denote colleges by C . Each college $C \in \mathcal{C} := \{1, 2\}$ has a capacity q_C which represents the maximum number of students that can be admitted to college C , where $q_C \geq 1$.

There are n students. We denote the set of all students by S . Since we suppose homogeneous preferences of students, we assume that each student has the cardinal utility v_C from college $C \in \{1, 2\}$, where $v_2 > v_1 > 0$. Thus we sometimes call college 2 the good college and college 1 the bad college. Each student's utility from not being assigned to any college is normalized to be 0. We assume that $q_1 + q_2 \leq n$.⁷

Each student is assigned to one college or no seat in any college by the mechanisms and the mechanisms take the efforts into account while deciding on their admissions.⁸ Each student $s \in S$ makes an effort e_s . The students are heterogeneous in terms of their abilities, and the abilities are their private information. More specifically, for each $s \in S$, $a_s \in [0, 1]$ denotes student s 's ability. Abilities are drawn identically and independently from the interval $[0, 1]$ according to a continuous distribution function F that is common knowledge. We assume that F has a continuous density $f = dF > 0$. For a student s with ability a_s , putting in an effort of e_s results in a disutility of $\frac{e_s}{a_s}$. Hence, the total utility of a student with ability a from making effort e is $v_C - e/a$ if she is assigned to college C , and $-e/a$ otherwise.

Before we move on to the analysis of the equilibrium of centralized and decentralized college admission mechanisms, we introduce some necessary notation.

2.1 Preliminary notation

First, for any continuous distribution T with density t , for $1 \leq k \leq m$, let $T_{k,m}$ denote the distribution of the k^{th} –(lowest) order statistics out of m independent random variables that are

⁶For a recent example for theory and experiments in school choice literature, see Chen and Kesten (2013).

⁷Many college admissions, including ones in Turkey and Japan, are competitive in the sense that total number of seats in colleges is smaller than the number of students who take the exams.

⁸In reality the performance in the entrance exams is only a noisy function of efforts. For simplicity, we assume that efforts completely determine the performance in the tests.

identically distributed according to T . That is,

$$T_{k,m}(a) := \sum_{j=k}^m \binom{m}{j} T(a)^j (1 - T(a))^{m-j}. \quad (1)$$

Moreover, let $t_{k,m}(\cdot)$ denote $T_{k,m}(\cdot)$'s density:

$$t_{k,m}(x) := \frac{d}{da} T_{k,m}(a) = \frac{m!}{(k-1)!(m-k)!} T(a)^{k-1} (1 - T(a))^{m-k} t(a). \quad (2)$$

For convenience, we let $T_{0,m}$ be a distribution with $T_{0,m}(a) = 1$ for all a , and $t_{0,m} \equiv dT_{0,m}/da = 0$.

Next, define the function $p_{j,k} : [0, 1] \rightarrow [0, 1]$ as follows: given $j, k \in \{0, 1, \dots, n\}$, for each $x \in [0, 1]$, define

$$p_{j,k}(x) := \binom{j+k}{j} x^j (1-x)^k. \quad (3)$$

The function $p_{j,k}(x)$ is interpreted as the probability that when there are $(j+k)$ students, j students are selected for one event with probability x and k students are selected for another event with probability $(1-x)$. Suppose that $p_{0,0}(x) = 1$ for all x . Note that with this definition, we can write

$$T_{k,m}(a) = \sum_{j=k}^m p_{j,m-j}(T(a)). \quad (4)$$

3 The Centralized College Admissions Mechanism (CCA)

In the centralized college admissions game, each student $s \in S$ simultaneously makes an effort e_s . Students with the top q_2 efforts are assigned to college 2 and students with the efforts from the top $(q_2 + 1)$ to $(q_1 + q_2)$ are assigned to college 1. The rest of the students are not assigned to any colleges.⁹ We now solve for the symmetric Bayesian Nash equilibrium of this game. The following proposition is a special case of the all-pay auction equilibrium which has been studied by Moldovanu and Sela (2001) and Moldovanu, Sela, and Shi (2012).

Proposition 1. *In CCA, there is a unique symmetric equilibrium β^C such that for each $a \in [0, 1]$,*

⁹In a setup with homogeneous student preferences, this game reflects how the Turkish college admission mechanism works. In the centralized test that the students take, since all students would put college 2 as their top choice and college 1 as their second top choice in their submitted preferences, the resulting assignment would be the same as the assignment described above. In a school choice context, this can be described as the following two-stage game. In the first stage, there is one contest where each student s simultaneously makes an effort e_s . The resulting effort profile $(e_s)_{s \in S}$ is used to construct a single priority profile \succ such that a student with a higher effort has a higher priority. In the second stage, students participate in the centralized deferred acceptance mechanism where colleges use the common priority \succ .

each student with ability a chooses an effort $\beta^C(a)$ according to

$$\beta^C(a) = \int_0^a x \{f_{n-q_2, n-1}(x) v_2 + (f_{n-q_1-q_2, n-1}(x) - f_{n-q_2, n-1}(x)) v_1\} dx.$$

where $f_{k,m}(\cdot)$ for $k \geq 1$ is defined in Equation (2) and $f_{0,m}(x)$ is defined to be 0 for all x .

Proof. Suppose that β^C is a symmetric equilibrium effort function that is strictly increasing. Consider a student with ability a who chooses an effort as if her ability is a' . Her expected utility is

$$v_2 F_{n-q_2, n-1}(a') + v_1 (F_{n-q_1-q_2, n-1}(a') - F_{n-q_2, n-1}(a')) - \frac{\beta^C(a')}{a}.$$

The first-order condition at $a' = a$ is

$$v_2 f_{n-q_2, n-1}(a) + v_1 (f_{n-q_1-q_2, n-1}(a) - f_{n-q_2, n-1}(a)) - \frac{[\beta^C(a)]'}{a} = 0.$$

Thus, by integration and as the boundary condition is $\beta^C(0) = 0$, we have

$$\beta^C(a) = \int_0^a x \{f_{n-q_2, n-1}(x) v_2 + (f_{n-q_1-q_2, n-1}(x) - f_{n-q_2, n-1}(x)) v_1\} dx.$$

The above strategy is the unique symmetric equilibrium candidate obtained via “the first-order approach” by requiring no benefit from local deviations. Standard arguments show that this is indeed an equilibrium by making sure that global deviations are not profitable (for instance, see Section 2.3 of Krishna (2002)). \square

4 The Decentralized College Admissions Mechanism (DCA)

In the decentralized college admissions game, each student s chooses one college C_s and an effort e_s simultaneously. Given the college choices of students $(C_s)_{s \in S}$ and efforts $(e_s)_{s \in S}$, each college C admits students with the top q_C effort levels among its set of applicants $(\{s \in S \mid C_s = C\})$.¹⁰

For this game, we solve for a symmetric Bayesian Nash equilibrium $(\gamma(\cdot), \beta^D(\cdot); c)$ where $c \in (0, 1)$ is a cutoff, $\gamma : [0, c] \rightarrow (0, 1)$ is the mixed strategy that represents the probability of lower ability students applying to college 1, and $\beta^D : [0, 1] \rightarrow \mathbb{R}$ is the continuous and strictly increasing effort function. Each student with type $a \in [0, c]$ chooses college 1 with probability $\gamma(a)$ (hence

¹⁰In a setup with homogeneous student preferences, this game reflects how the Japanese college admissions mechanism works: all public colleges hold their own tests and accept the top performers among the students *who take their tests*. In school choice context, this can be described as the following two-stage game. In the first stage, students simultaneously choose which school to apply to, and without knowing how many other students have applied, they also choose their effort level. For each school $C \in \{1, 2\}$, the resulting effort profile $(e_s)_{\{s \in S \mid C_s = C\}}$ is used to construct one priority profile \succ_C such that a student with a higher effort has a higher priority. In the second stage, students participate in two separate deferred acceptance mechanisms where each college C uses the priority \succ_C .

chooses college 2 with probability $1 - \gamma(a)$, and makes effort $\beta^D(a)$. Each student with type $a \in (c, 1]$ chooses college 2 for sure, and makes effort $\beta^D(a)$.¹¹

We now move on to the derivation of symmetric Bayesian Nash equilibrium. Let a symmetric strategy profile $(\gamma(\cdot), \beta(\cdot); c)$ be given. For this strategy profile, the ex-ante probability that a student applies to college 1 is $\int_0^c \gamma(x)f(x)dx$, while the probability that a student applies to college 2 is $1 - \int_0^c \gamma(x)f(x)dx$. Let us define a function $\pi : [0, c] \rightarrow [0, 1]$ that represents the ex-ante probability that a student has a type less than a and she applies to college 1:

$$\pi(a) := \int_0^a \gamma(x)f(x)dx. \quad (5)$$

With this definition, the ex-ante probability that a student applies to college 1 is $\pi(c)$, while the probability that a student applies to college 2 is $1 - \pi(c)$. Moreover, $p_{m,k}(\pi(c))$ is the probability that m students apply to college 1 and k students apply to college 2 where $p_{m,k}(\cdot)$ is given in Equation (3) and $\pi(\cdot)$ is given in Equation (5).

Next, we define $G(\cdot) : [0, c] \rightarrow [0, 1]$, where $G(a)$ is the probability that a type is less than or equal to a , conditional on the event that she applies to college 1. That is,

$$G(a) := \frac{\pi(a)}{\pi(c)}.$$

Moreover let $g(\cdot)$ denote $G(\cdot)$'s density. $G_{k,m}$ is the distribution of the k^{th} -order statistics out of m independent random variables that are identically distributed according to G as in equations (1) and (4). Also, $g_{k,m}(\cdot)$ denotes $G_{k,m}(\cdot)$'s density.

Similarly, let us define $H(\cdot) : [0, 1] \rightarrow [0, 1]$, where $H(a)$ is the probability that a type is less than or equal to a , conditional on the event that she applies to college 2. That is, for $a \in [0, 1]$,

$$H(a) = \begin{cases} \frac{F(a) - \pi(a)}{1 - \pi(c)} & \text{if } a \in [0, c], \\ \frac{F(a) - \pi(c)}{1 - \pi(c)} & \text{if } a \in [c, 1]. \end{cases}$$

Moreover, let $h(\cdot)$ denote $H(\cdot)$'s density. Note that h is continuous but is not differentiable at c . Let $H_{k,m}$ be the distribution of the k^{th} -order statistics out of m independent random variables distributed according to H as in equations (1) and (4). Also, $h_{k,m}(\cdot)$ denotes $H_{k,m}(\cdot)$'s density.

¹¹A natural equilibrium candidate is to have a cutoff $c \in (0, 1)$, students with abilities in $[0, c)$ to apply to college 1, and students with abilities in $[c, 1]$ to apply to college 2. It turns out that we cannot have an equilibrium of this kind. In such an equilibrium, (i) type c has to be indifferent between applying to college 1 or college 2, (ii) type c 's effort is strictly positive in case of applying to college 1, and 0 while applying to college 2, hence there is a discontinuity in the effort function. These two conditions together imply that a type $c + \epsilon$ student would benefit from mimicking a type $c - \epsilon$ student for a small enough ϵ . Formal arguments resulting in the nonexistence result are available from the authors upon request. Therefore, we have to have some students using mixed strategies while choosing which college to apply to. Derivations show that in equilibrium, lower ability students would use mixed strategies, while the higher ability students are certain to apply to the better school.

We are now ready to state the main result of this section, which characterizes the *unique* symmetric Bayesian Nash equilibrium¹² of the decentralized college admissions mechanism. The sketch of the proof follows the Theorem, whereas the more technical part of the proof is relegated to Appendix B.

Theorem 1. *In DCA, there is a unique symmetric equilibrium $(\gamma, \beta^D; c)$ where a student with type $a \in [0, c]$ chooses college 1 with probability $\gamma(a)$ and makes effort $\beta^D(a)$; and a student with type $a \in [c, 1]$ chooses college 2 for sure and makes effort $\beta^D(a)$. Specifically,*

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx.$$

The equilibrium cutoff c and the mixed strategies $\gamma(\cdot)$ are determined by the following four requirements:

(i) $\pi(c)$ uniquely solves the following equation for x

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x).$$

(ii) Given $\pi(c)$, c uniquely solves the following equation for x

$$v_1 = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{F(x) - \pi(c)}{1 - \pi(c)} \right).$$

(iii) Given $\pi(c)$ and c , for each $a \in [0, c]$, $\pi(a)$ uniquely solves the following equation for $x(a)$

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{F(a) - x(a)}{1 - \pi(c)} \right) = v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j} \left(\frac{x(a)}{\pi(c)} \right).$$

(iv) Finally, for each $a \in [0, c]$, $\gamma(a)$ is given by

$$\gamma(a) = \frac{\pi(c)B(a)}{(1 - \pi(c))A(a) + \pi(c)B(a)} \in (0, 1),$$

¹²More specifically, we characterize the unique equilibrium in which (i) students use a mixed strategy while deciding which college to apply to, and (ii) effort levels are independent of college choice and monotone increasing in abilities.

where

$$A(a) := v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) m p_{m-q_1,q_1-1} \left(\frac{\pi(a)}{\pi(c)} \right),$$

$$B(a) := v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) m p_{m-q_2,q_2-1} \left(\frac{F(a) - \pi(a)}{1 - \pi(c)} \right).$$

Proof. Suppose that each student with type $a \in [0, 1]$ follows a strictly increasing effort function β^D and a type $a \in [0, c]$ chooses college 1 with probability $\gamma(a) \in (0, 1)$, and a type in $(c, 1]$ chooses college 2 for sure.

We first show how to obtain the equilibrium cutoff c and the mixed strategy function γ . A necessary condition for this to be an equilibrium is that each type $a \in [0, c]$ has to be indifferent between applying to college 1 or 2. Thus, for all $a \in [0, c]$,

$$v_1 \left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) G_{m-q_1+1,m}(a) \right)$$

$$= v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a) \right). \quad (6)$$

The left-hand side is the expected utility of applying to college 1, while the right-hand side is the expected utility of applying to college 2. To see this, note that $\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c))$ and $\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c))$ are the probabilities that there are no more than $(q_1 - 1)$ and $(q_2 - 1)$ applicants in colleges 1 and 2, respectively. For $m \geq q_1$, $p_{m,n-m-1}(\pi(c)) G_{m-q_1+1,m}(a)$ is the probability of getting a seat in college 1 with effort a when there are m other applicants in college 1. Similarly, for $m \geq q_2$, $p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a)$ is the probability of getting a seat in college 2 with effort a , when there are m other applicants in college 2.

Note that we have

$$G_{m-q_1+1,m}(a) = \sum_{j=m-q_1+1}^m p_{j,m-j} \left(\frac{\pi(a)}{\pi(c)} \right) \quad \text{and} \quad H_{m-q_2+1,m}(a) = \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{F(a) - \pi(a)}{1 - \pi(c)} \right)$$

for all $a \in [0, c]$. The equation (6) at $a = 0$ and $a = c$ can hence be written as

$$v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)), \quad \text{and} \quad (7)$$

$$v_1 = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{F(c) - \pi(c)}{1 - \pi(c)} \right), \quad (8)$$

respectively.

We show in Appendix B that there is a unique $\pi(c)$ that satisfies Equation (7), and that given $\pi(c)$, the only unknown c via $F(c)$ in Equation (8) is uniquely determined. Moreover, using (7), we can rewrite Equation (6) as

$$v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j} \left(\frac{\pi(a)}{\pi(c)} \right) = v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{F(a) - \pi(a)}{1 - \pi(c)} \right), \quad (9)$$

for all $a \in [0, c]$. In Appendix B, we show that given $\pi(c)$ and c , for each $a \in [0, c]$, there is a unique $\pi(a)$ that satisfies Equation (9) and, moreover, that we can get the mixed strategy function $\gamma(a)$ by differentiating Equation (9).

Finally we derive the unique symmetric effort function β^D by taking a “first-order approach” in terms of $G(\cdot)$ and $H(\cdot)$ which are determined by the equilibrium cutoff c and the mixed strategy function γ . Consider a student with type $a \in [0, c]$. A necessary condition for the strategy to be an equilibrium is that she does not want to mimic any other type a' in $[0, c]$. Her utility maximization problem is given by

$$\max_{a' \in [0, c]} v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) H_{m-q_2+1,m}(a') \right) - \frac{\beta^D(a')}{a}.$$

where the indifference condition (6) is used to calculate the expected utility.¹³ The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition (which is $\beta^D(0) = 0$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) h_{m-q_2+1,m}(x) dx$$

for all $a \in [0, c]$

¹³Equivalently, we can write the maximization problem as

$$\max_{a' \in [0, c]} v_1 \left(\sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) + \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) G_{m-q_1+1,m}(a) \right) - \frac{\beta^D(a')}{a},$$

With the same procedure, this gives the equivalent solution as

$$\beta^D(a) = v_1 \int_0^a x \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) g_{m-q_1+1,m}(x) dx$$

for each $a \in [0, c]$.

Next, consider a student with type $a \in [c, 1]$. A necessary condition is that she does not want to mimic any other type a' in $[c, 1]$. Her utility maximization problem is then

$$\max_{a' \in [c, 1]} v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1, m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1, m}(\pi(c)) H_{m-q_2+1, m}(a') \right) - \frac{\beta^D(a')}{a}.$$

Note that although the objective function is the same for types in $[0, c]$ and $[c, 1]$, it is not differentiable at the cutoff c . The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence,

$$v_2 \sum_{m=q_2}^{n-1} p_{n-m-1, m}(\pi(c)) h_{m-q_2+1, m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition of continuity (which is $\beta^D(c) = v_2 \int_0^c x \sum_{m=q_2}^{n-1} p_{n-m-1, m}(\pi(c)) h_{m-q_2+1, m}(x) dx$), we obtain

$$\beta^D(a) = v_2 \int_0^a x \sum_{m=q_2}^{n-1} p_{n-m-1, m}(\pi(c)) h_{m-q_2+1, m}(x) dx$$

for each $a \in [c, 1]$.

To complete the proof, we need to show that not only local deviations, but also global deviations cannot be profitable. In Appendix B.2, we do that and hence show that the uniquely derived symmetric strategy $(\gamma, \beta^D; c)$ is indeed an equilibrium. \square

5 Comparisons

As illustrated in sections 3 and 4, the two mechanisms result in different equilibria. It is therefore natural to ask how the two equilibria compare in terms of interim student welfare. We denote by $EU^C(a)$ and $EU^D(a)$ the expected utility of a student with ability a under CCA and DCA, respectively.

Our first result concerns the preference of low-ability students.

Proposition 2. *Low-ability students prefer DCA to CCA if and only if $n > q_1 + q_2$.*

Proof. First, let us consider the case of $n > q_1 + q_2$. For this case it is not difficult to see that $EU^C(0) = 0$ (because the probability of being assigned to any college is zero), and $EU^D(0) > 0$ (because with a positive probability, type 0 will be assigned to a college). Since the utility functions are continuous, we can then see that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^D(x) > EU^C(x)$.

Next, let us consider the case of $n = q_1 + q_2$. For this case, we have $EU^C(0) = v_1$. This is because with probability 1, type 0 will be assigned to college 1 by exerting 0 effort. Moreover, we

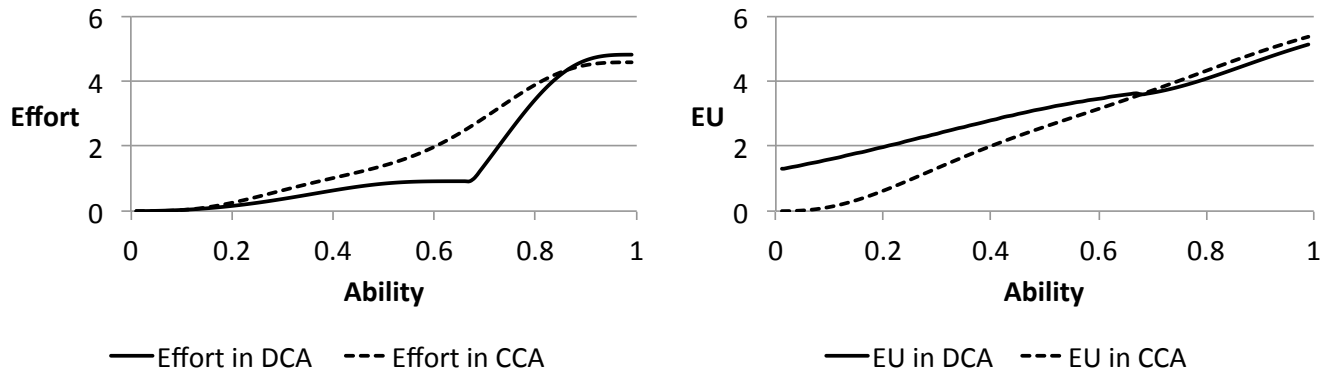


Figure 1: Efforts (left) and expected utility (right) under CCA and DCA

Note: The figures were created with the help of simulations for the following parameters: $n = 12$, $(q_1, q_2) = (5, 4)$, and $(v_1, v_2) = (5, 20)$. The equilibrium cutoff under DCA is calculated as $c = 0.675$.

have $EU^D(0) < v_1$. This is because type 0 should be indifferent between applying to college 1 and college 2, and in the case of applying to college 1, the probability of getting assigned to college 1 is strictly smaller than 1. Since the utility functions are continuous, we can then see that there exists an $\epsilon > 0$ such that for all $x \in [0, \epsilon]$, we have $EU^C(x) > EU^D(x)$. \square

Intuitively, when the seats are over-demanded (i.e., when $n > q_1 + q_2$), very low-ability students have almost no chance of getting a seat in CCA, whereas their probability of getting a seat in DCA is bounded away from zero. Hence they prefer DCA.

Although this result merely shows that only students in the neighborhood of type 0 need to have these kinds of preferences, explicit equilibrium calculations for many examples (such as the markets we study in our experiments) result in a significant proportion of low-ability students preferring DCA. We provide an explicit depiction of equilibrium effort levels and interim expected utilities for a specific example in Figure 1.

Moreover, we establish the reverse ranking for the high-ability students. That is, the high-ability students prefer CCA in the following single-crossing sense: if a student who applies to college 2 in DCA prefers CCA to DCA, then all higher ability students have the same preference ranking.

Proposition 3. *Let c be the equilibrium cutoff in DCA. We have (i) if $EU^C(a) \geq EU^D(a)$ for some $a > c$, then $EU^C(a') > EU^D(a')$ for all $a' > a$, and (ii) if $EU^C(a) < EU^D(a)$ for some $a > c$, then $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$.*

Proof. Let us define

$$\begin{aligned} K(a) &\equiv v_2 F_{n-q_2, n-1}(a), \\ L(a) &\equiv v_1 (F_{n-q_1-q_2, n-1}(a) - F_{n-q_2, n-1}(a)), \\ M(a) &\equiv K(a) + L(a), \\ N(a) &= v_2 \left(\sum_{m=0}^{q_2-1} p_{n-m-1, m}(\pi(c)) + \sum_{m=q_2}^{n-1} p_{n-m-1, m}(\pi(c)) H_{m-q_2+1, m}(a) \right). \end{aligned}$$

Then we have

$$EU^C(a) = M(a) - \frac{\int_0^a M'(x) x dx}{a}.$$

By integration by parts, we obtain

$$EU^C(a) = \frac{\int_0^a M(x) dx}{a}.$$

Similarly,

$$EU^D(a) = N(a) - \frac{\int_0^a N'(x) x dx}{a},$$

and by integration by parts, we obtain

$$EU^D(a) = \frac{\int_0^a N(x) dx}{a}.$$

Note that, for $a > c$, we have

$$N(a) = K(a).$$

This is because students whose ability levels are greater than c apply to college 2 in DCA, and therefore a seat is granted to a student with ability level $a > c$ if and only if the number of students with ability levels greater than a is not greater than q_2 . This is the same condition in CCA, which is given by the expression $K(a)!$. (Also note that we have $N(a) \neq K(a)$ for $a < c$, in fact we have $N(a) > K(a)$, but this is irrelevant for what follows.)

Now, for any $a > c$, we obtain

$$\begin{aligned} \frac{d}{da} (aEU^C(a)) &= M(a) \\ &= K(a) + L(a) \end{aligned}$$

and

$$\begin{aligned} \frac{d}{da} (aEU^D(a)) &= N(a) \\ &= K(a). \end{aligned}$$

Since $L(a) > 0$, for any $a > c$, we have

$$\frac{d}{da} (aEU^C(a)) > \frac{d}{da} (aEU^D(a)),$$

or

$$EU^C(a) + a\frac{d}{da}EU^C(a) > EU^D(a) + a\frac{d}{da}EU^D(a).$$

This means that for any $a > c$, whenever $EU^C(a) = EU^D(a)$, we have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. Then we can conclude that once $EU^C(a)$ is higher than $EU^D(a)$, it cannot cut through $EU^D(a)$ from above to below and $EU^C(a)$ always stays above $EU^D(a)$. To see this suppose $EU^C(a) > EU^D(a)$ and $EU^C(a') < EU^D(a')$ for some $a' > a > c$, then (since both $EU^C(a)$ and $EU^D(a)$ are continuously differentiable) there exists $a'' \in (a, a')$ such that $EU^C(a'') = EU^D(a'')$ and $\frac{d}{da}EU^C(a'') < \frac{d}{da}EU^D(a'')$, a contradiction. Hence (i) is satisfied. Moreover, (ii) is obviously satisfied since whenever $EU^C(a) < EU^D(a)$, we have to have $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$. \square

Intuitively, since high-ability students (i) can only get a seat in the good college in DCA whereas they can get a seat in both the good and the bad college in CCA, and (ii) their equilibrium probability of getting a seat in the good college is the same across the two mechanisms, they prefer CCA.

One may also wonder if there is a general “ex ante” utility ranking between DCA and CCA. It turns out that one can find examples where either DCA or CCA result in higher ex ante utility (or social welfare).¹⁴

6 The Case of l Colleges

Let us consider l colleges, $1, \dots, l$, where each college k has the capacity $q_k > 0$ and each student gets the utility of v_k from attending college k ($v_l > v_{l-1} > \dots > v_2 > v_1 > 0$).

We conjecture¹⁵ that in the decentralized mechanism there will be a symmetric Bayesian Nash equilibrium $((\gamma_k)_{k=1}^l, \beta^D, (c_k)_{k=0}^l)$: (i) c_0, \dots, c_l are cutoffs such that $0 = c_0 < c_1 < \dots < c_{l-1} < c_l = 1$; (ii) β^D is an effort function where each student with ability a makes an effort level of $\beta^D(a)$; (iii) $\gamma_1, \dots, \gamma_l$ are mixed strategies such that for each $k \in \{1, \dots, l-1\}$, each student with ability $a \in [c_{k-1}, c_k)$ applies to college k with probability $\gamma_k(a)$ and college $k+1$ with probability $1 - \gamma_k(a)$. Moreover, each student with ability $a \in [c_{l-1}, 1]$ applies to college l , equivalently, $\gamma_l(a) = 1$. The equilibrium effort levels can be identified as follows.

Let $k \in \{1, \dots, l\}$ be given. Let $\pi^k(a)$ denote the ex-ante probability that a student has a type less than or equal to a and she applies to college k . Then, $\pi^1(a) = \int_0^a \gamma_1(x) dF(x)$. For

¹⁴Specific examples are available from the authors upon request.

¹⁵As explained below, the strategies are not formally shown to be an equilibrium since we do not have a proof to show that global deviations are not profitable.

$k \in \{2, \dots, l\}$ and $a \in [c_{k-2}, c_k]$,

$$\pi^k(a) = \begin{cases} \int_{c_{k-2}}^a (1 - \gamma_{k-1}(x)) dF(x) & \text{if } a \leq c_{k-1}, \\ \int_{c_{k-2}}^{c_{k-1}} (1 - \gamma_{k-1}(x)) dF(x) + \int_{c_{k-1}}^a \gamma_k(x) dF(x) & \text{if } a \geq c_{k-1}. \end{cases}$$

We define H^k to be the probability that a type is less than or equal to a , conditional on the event that she applies to college k :

$$H^k(a) = \frac{\pi^k(a)}{\pi^k(c_k)}.$$

In this equilibrium, each student with ability $a \in [c_{k-1}, c_k]$ exerts an effort of

$$\beta^D(a) = \beta^D(c_{k-1}) + \int_{c_{k-1}}^a x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h_{m-q_k+1,m}^k(x) dx$$

where $\beta^D(0) = 0$ and $h_{m-q_k+1,m}^k$ is the density of $H_{m-q_k+1,m}^k$. Similar to Theorem 1, it is possible to determine the formulation for cutoffs c_1, \dots, c_{l-1} and mixed strategies $\gamma_1, \dots, \gamma_l$ using the indifference conditions (See the Appendix C). This set of strategies can be shown to satisfy immunity for “local deviations,” but prohibitively tedious arguments to check for immunity to global deviations (as we have done in the Appendix B) prevent us from formally proving that it is indeed an equilibrium.

By supposing an equilibrium of this kind, we can actually show that propositions 2 and 3 hold for l colleges. Proposition 2 trivially holds, as students with the lowest ability levels get zero utility from CCA and strictly positive utility from DCA. We can also argue that Proposition 3 holds since the students with ability levels $a \in [c_{l-1}, 1]$ apply to college l only. This can be observed by noting that a seat is granted to these students in college k if and only if the number of students with ability levels greater than a is no greater than q_l , which is the same condition in CCA. Hence, even in this more general setup of l colleges, we can argue that low-ability students prefer DCA whereas high-ability students prefer CCA.

7 The Experiment

In this section, we present an experiment on college admissions with entrance exams. It is designed to test the results of the model and generate further insights into the performance of the centralized (CCA) and the decentralized college admissions mechanism (DCA). In particular, we check which of the two mechanisms leads to higher student efforts and welfare in the experiment. We investigate individual effort choices by the students in the two mechanisms as well as their choice of college in DCA.

7.1 Design of the experiment

In the experiment, there are two colleges, college 1 (the bad college) and college 2 (the good college). There are 12 students who apply for positions, and these students differ with respect to their ability. At the beginning, every student learns her ability a_s . The ability of each student is drawn from the uniform distribution over the interval of 0 to 100. Students have to choose an effort level e_s that determines their success in the application process. The cost of effort is given by $\frac{e_s}{a_s}$.

In the centralized college admissions mechanism (CCA), all students simultaneously choose an effort level. Then the computer determines the matching by admitting the students with the highest effort levels to college 2 up to its capacity q_2 and the next best students, i.e., from rank $q_1 + 1$ to rank $q_1 + q_2$, to college 1. All other students are unassigned.

In the decentralized college admissions mechanism (DCA), the students simultaneously decide not only on their effort level but also on which of the colleges to apply to. The computer determines the matching by assigning the students with the highest effort among those who have applied to college C , up to its capacity q_C .

We implemented five different markets that differ with respect to the total number of open slots ($q_1 + q_2$), the number of slots at each college (q_1 and q_2) as well as the value of the colleges for the students (v_1 and v_2). This allows us to investigate behavior under very different market conditions. Most relevant from the point of view of the theoretical predictions, we can compare outcomes in markets where the number of students is equal to the number of seats (markets 1 and 4) to markets with more students than seats (markets 2, 3, and 5). The parameters in each market were chosen so as to generate clear-cut predictions regarding the two main outcome variables, effort and the interim expected utility of each student. In each of the first four markets, one mechanism dominates the other in one of the two outcome variables. The fifth market is designed to make the two mechanisms as similar as possible.

In order to provide a valid comparison of the observed average effort and utility levels in the markets where there is no dominance relationship, i.e., the cells in Table 1 for which the predicted difference depends on the ability of the applicant, we compute the equilibrium effort and utility levels for the realizations of abilities in our experimental markets. We then take expected values given the realized abilities. Table 1 provides an overview of the five markets together with the theoretical predictions regarding the difference between CCA and DCA.

We employed a between-subjects design. Students were randomly assigned either to the treatment with CCA or the treatment with DCA. In each treatment, subjects played 15 rounds with one market per round. Each of the five different markets was played three times by every participant, and abilities were drawn randomly for every round. These draws were independent, and each ability was equally likely. We employed the same randomly drawn ability profiles in both treatments in order to make them as comparable as possible. Markets were played in blocks: first all five

Table 1: Overview of market characteristics

	Number of seats at [Value of]		Predicted utility higher	Predicted effort higher
	college 2	college 1		
Market 1	6 [2000]	6 [1000]	CCA	depends; DCA in expectation
Market 2	2 [2000]	2 [1000]	DCA	no diff. in expectation
Market 3	2 [2000]	8 [1000]	depends; DCA in expectation	CCA
Market 4	3 [2000]	9 [1800]	CCA	DCA
Market 5	9 [2000]	1 [1000]	no diff. in expectation	no diff. in expectation

Notes: In columns 4 and 5, one of the two mechanisms sometimes dominates the other for all students, but the ranking of the mechanisms can also depend on the students' ability.

markets were played in a random order once, then all five markets were played in a random order for a second time, and then again randomly ordered for the last time. We chose this sequence of markets in order to ensure that the level of experience does not vary across markets. Participants faced a new situation in every round as they never played the same market with the same ability twice. They received feedback about their allocation and the points they earned after every round.

At the beginning of the experiment, students received an endowment of 2,200 points. At the end of the experiment, one of the 15 rounds was randomly selected for payment. The points earned in this round plus the 2,200 endowment points were paid out in Euro with an exchange rate of 0.5 cents per point. The experiment lasted 90 minutes, and the average earnings per subject were EUR 14.10.

The experiment was run at the experimental economics lab at the Technical University Berlin. We recruited student subjects from our pool with the help of ORSEE by Greiner (2004). The experiments were programmed in z-Tree, see Fischbacher (2007). For each of the two treatments, CCA and DCA, independent sessions were carried out. Each session consisted of 24 participants that were split into two matching groups of 12 for the entire session. In total, six sessions were conducted, that is, three sessions per treatment, with each session consisting of two independent matching groups of 12 participants. Thus, we end up with six fully independent matching groups and 72 participants per treatment.

In the beginning of the experiment, printed instructions were given to the participants (see Appendix D). Participants were informed that the experiment was about the study of decision making, and that their payoff depended on their own decisions and the decisions of the other participants. The instructions were identical for all participants of a treatment, explaining in detail the experimental setting. Questions were answered in private. After reading the instructions, all individuals participated in a quiz to make sure that everybody understood the main features of the experiment.

7.2 Experimental results

We first present the aggregate results in order to compare the two mechanisms. In a second step, we study behavior in the two mechanisms separately to compare it to the point predictions and to shed light on the reasons for the aggregate findings. All results we report on are significant at the 5% level.

7.2.1 Treatment comparisons: Aggregate results

We compare the two college admission mechanisms with respect to three properties, summarized in results 1 to 3. The first comparison concerns the expected utility of students in the two mechanisms, which is equal to the expected number of points earned, due to the assumption of risk neutrality. Second, we investigate whether one of the mechanisms leads to higher effort levels by the students than the other mechanism. And the third aspect we focus on is whether individuals of different ability prefer different mechanisms.

Result 1 (Expected utility): In markets 1 and 4, where $n = q_1 + q_2$, the average utility of students in CCA is higher than in DCA, as predicted by the theory. In markets 2 and 3, the average utility of students in DCA is not higher than in CCA, in contrast to the theoretical predictions. In market 5, there is no significant difference both in theory and in the data.

Support. Table 2 presents the average number of points or the average utility of the participants in the two different mechanisms in all five markets. The third column displays the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of *equilibrium* utilities and efforts, based on the realized draw of abilities. Thus, in markets 1 to 4, we expect that level of utility in the two mechanisms is significantly different. The last column in the table provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the *observed* number of points earned in the two mechanisms.

Table 2: Average utility

Market	Utility higher for all students (predicted)	Average utility higher for realized types (predicted)	Average utility in CCA (observed)	Average utility in DCA (observed)	Observed utilities different in CCA and DCA
1	CCA	CCA, 0.00	1223	1021	0.01
2	DCA	DCA, 0.02	111	86	0.75
3	depends; DCA in expectation	DCA, 0.00	603	576	0.75
4	CCA	CCA, 0.00	1058	747	0.00
5	no diff. in expectation	no diff., 0.63	1183	1160	0.63

Notes: Columns 3 and 6 show the p-values of the Wilcoxon rank-sum test for equality of the distributions.

The equilibrium predictions for the comparison of utilities of students in markets 1 and 4 are consistent with the experimental data, as the average utility in CCA is significantly higher in both markets. Thus, with an equal number of applicants and seats, CCA is preferable to DCA if

the goal is to maximize the utility of the students. This is due to the potential miscoordination of applicants in DCA. We fail to observe the superiority of DCA in both markets where this is predicted, namely markets 2 and 3. The relationship is even reversed, with the average utility being higher in CCA than in DCA in both markets.

Result 2 (Effort levels): In markets 1 and 4, where $n = q_1 + q_2$, the average effort level of students in DCA is higher than in CCA. This is in line with the predictions. In market 3, the average effort levels of students in CCA are not significantly higher than in DCA, in contrast to the theoretical prediction. In markets 2 and 5, there is no difference in effort between the two mechanisms both in theory and in the data.

Support. Table 3 presents the average effort levels of the participants by different mechanisms and markets. Analogously to Table 2, the third column shows the results of the Wilcoxon rank sum test of the equilibrium efforts based on the realized draw of abilities. We expect effort to differ significantly between the two mechanisms only in markets 2 and 3 (with a marginally significant difference in market 1). The last column provides the p-values for the two-sided Wilcoxon rank-sum test for the equality of distributions of the observed effort levels in the two mechanisms. The equilibrium predictions regarding the comparison of efforts in markets 1 and 4 are confirmed by the data in that effort is higher in DCA. In market 3 average efforts are higher CCA than in DCA as predicted, but the difference is not significant.

Table 3: Average effort

Market	Effort higher for all students (predicted)	Average effort higher for realized types (predicted)	Average effort in CCA (observed)	Average effort in DCA (observed)	Observed efforts different in CCA and DCA
1	depends; DCA in expectation	DCA, 0.06	276	362	0.04
2	no diff. in expectation	no diff., 0.15	389	410	0.75
3	CCA	CCA, 0.00	397	354	0.42
4	DCA	DCA, 0.00	191	340	0.02
5	no diff. in expectation	no diff., 0.75	400	395	1.00

Notes: Columns 3 and 6 show the p-values of the Wilcoxon rank-sum test for equality of the distributions.

Taking together results 1 and 2, we observe that in markets without a shortage of seats (market 1 and market 4) students are on average better off in CCA where they exert less effort. In market 5 the results are also in line with the theoretical predictions with almost identical effort and expected utility levels in both mechanisms. In the two remaining markets with a surplus of students over seats, markets 2 and 3, the results contradict the theory. Markets 2 and 3 should lead to a higher average utility of the students in DCA than in CCA, which cannot be observed in the lab. Therefore, the overall results suggest that with respect to the utility of students, CCA performs better than predicted relative to DCA.

Next we turn to the question whether students of different abilities prefer different mechanisms by providing an experimental test of propositions 2 and 3. According to Proposition 2 low-ability

students prefer DCA over CCA if there are more applicants than seats in the market, as in our markets 2, 3, and 5. Proposition 3 implies that if any student prefers CCA over DCA, then all students with a higher ability must also prefer CCA. (Remember that in markets 1 and 4, all students prefer CCA, and we therefore do not consider these markets here.)

Result 3 (Expected utility of low- and high-ability students): In markets 2 and 3, the average utilities of students with low abilities are higher in DCA, and the average utilities of students with high abilities are higher in CCA. There is no difference in the average utilities of students in DCA and CCA in market 5.

Support: Table 4 presents the regression results of the students' utility on the 10% ability quantiles and the dummies for the interaction of each quantile and the DCA for market 2 and market 3. The significance of the dummy variables for the interaction of the DCA and a quantile reflects the significance of the treatment difference for the corresponding 10% ability quantile. Coefficients for the interactions of the first to fourth quantiles (i.e., the students with the lowest ability) and the DCA are positive, and two of them are significantly different from zero. Thus, the low-ability students have on average a higher utility in DCA in markets 2 and 3. Coefficients for the other quantiles are negative, and are significant for the seventh and tenth 10% quantiles. Thus, high-ability students have, on average, a lower utility in DCA than in CCA. This confirms the single-crossing property of Proposition 3. Overall, the results of markets 2 and 3 lend support to propositions 2 and 3. Note that market 5 which we constructed as a control to generate approximately the same outcome for CCA and DCA, does not yield significant differences in the expected utility for high- and low-ability students.

7.2.2 Point predictions regarding individual behavior

Next we investigate the individual behavior of subjects in each mechanism separately. In particular, we test the point predictions of the theory regarding the effort levels in CCA and DCA as well as the choice between college 1 and college 2 in DCA. This will help to understand the results regarding the comparison of the two mechanisms, in particular the relatively poor performance of DCA with respect to student utility.

Figure 2 depicts the efforts of individuals, the kernel regression estimation of efforts, and the equilibrium predictions for each of the markets and mechanisms. All 10 panels for the 10 markets show that the kernel of effort increases in ability. Moreover, the observed effort levels often lie above the predicted values.

Result 4 (Individual efforts): Individual efforts in the experiments differ from the equilibrium efforts in eight out of 10 markets. In all 10 markets average efforts are greater than average equilibrium efforts. This overexertion of effort is significant in all five markets in DCA and in three out of five markets in CCA. The observed effort levels differ from random behavior, and equilibrium efforts have predictive power for the observed effort levels in both mechanisms.

Table 4: Utility differences across ability quantiles

Variable	Coefficient (Std. Err.)
10% ability quantiles	49.008*** (8.069)
1st quantile in DCA	98.812 (83.255)
2nd quantile in DCA	294.889*** (76.675)
3rd quantile in DCA	234.895*** (73.484)
4th quantile in DCA	57.848 (86.449)
5th quantile in DCA	-79.696 (93.920)
6th quantile in DCA	-60.945 (92.340)
7th quantile in DCA	-278.143*** (91.047)
8th quantile in DCA	-103.370 (112.019)
9th quantile in DCA	-190.702 (118.914)
10th quantile in DCA	-186.753** (110.123)
Intercept	80.770* (45.231)
N	
	864
R ²	
	0.047
F _(11,852)	
	3.819

Notes: OLS estimation of utility based on clustered robust standard errors at the subject level. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level.

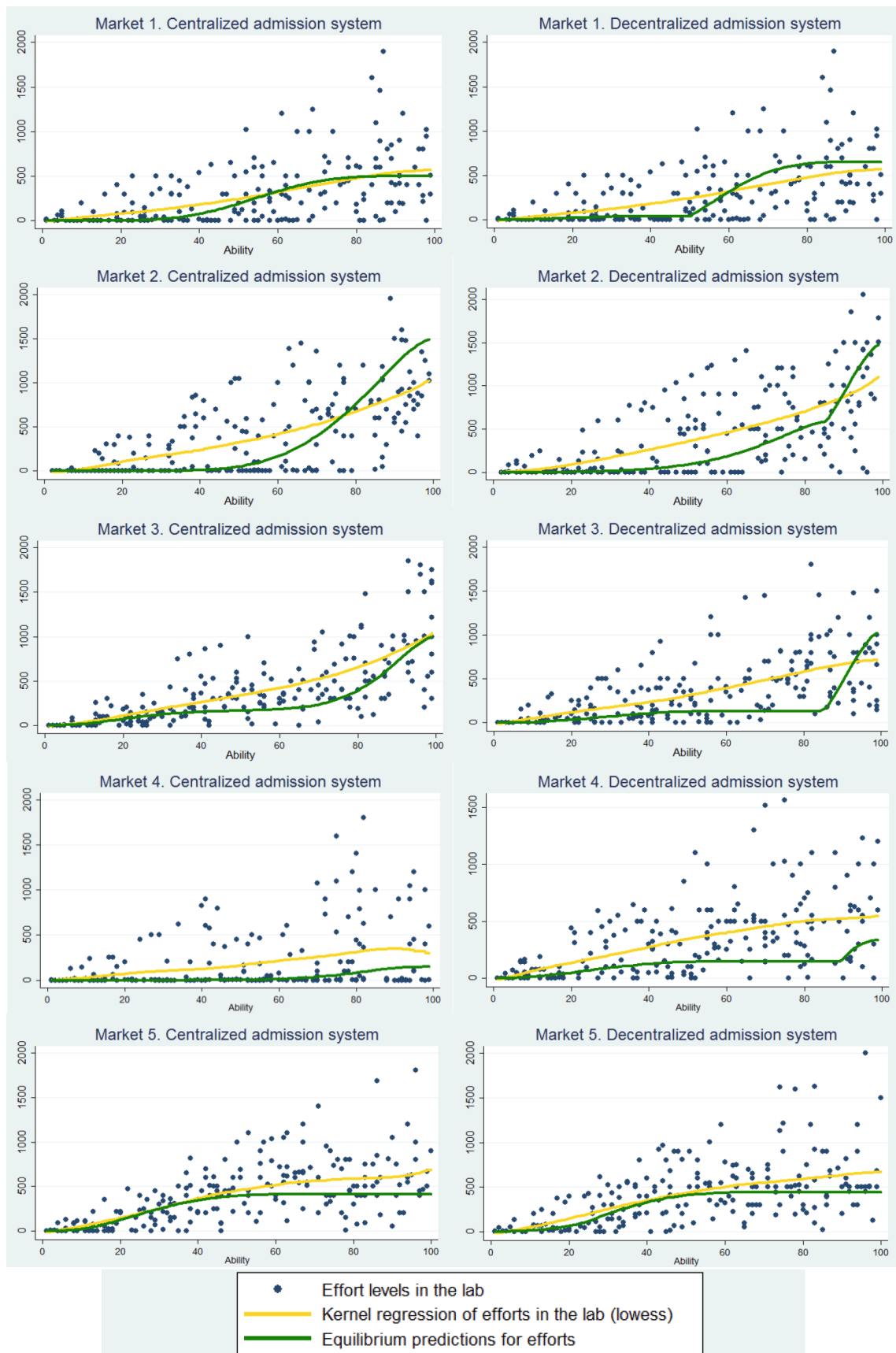


Figure 2: Individual efforts by ability

Table 5: Individual efforts

	Average observed efforts (1)	Average equilibrium efforts (2)	Average random efforts (3)	p-value obs.=pred. (4)	p-value obs.=rand. (5)
CCA					
Market 1	276	230	548	0.41	0.00
Market 2	389	364	567	0.74	0.00
Market 3	397	280	572	0.00	0.00
Market 4	191	35	553	0.00	0.00
Market 5	400	305	551	0.00	0.00
DCA					
Market 1	362	262	548	0.00	0.00
Market 2	410	309	567	0.00	0.00
Market 3	354	195	572	0.00	0.00
Market 4	340	125	553	0.00	0.00
Market 5	395	307	551	0.00	0.00

Support: In all markets and mechanisms, average effort levels are higher than predicted, as can be taken from a comparison of the first two columns in Table 5. Column (4) provides the p-values of the Wilcoxon matched-pairs signed-rank test for the equality of observed and equilibrium efforts by markets and mechanisms. In CCA the difference is significant for three out of five markets (market 3, 4, and 5) while in DCA it is significant for all five markets. Thus DCA leads to significant overexertion in more markets than CCA. One possible intuition for this finding is that the uncertainty is higher under DCA where students need to coordinate on the colleges, which leads to higher efforts.

Next, we compare observed behavior to random choices. As the ability level of a student determines her possible set of effort choices, random choices will differ for different ability types. Thus, we define the random choice as the choice of the effort in the middle of the interval of all feasible efforts of an applicant, see column (3). The behavior of subjects is significantly different from the random prediction in all markets for both mechanisms as can be taken from the p-values of the Wilcoxon matched-pairs rank-sum test for the equality of observed and random efforts in the last column of Table 5.

We also find that in spite of the negative results regarding the point predictions, the equilibrium effort levels have significant predictive power. This emerges from an OLS estimation of observed efforts based on clustered robust standard errors at the level of matching groups, presented in Table 6. Moreover, there is no significant difference with respect to the predictive power of the equilibrium in the different admission systems (as the predictions for CCA and the dummy variable for CCA are both not significant).

As a final step, we investigate the choice of participants to apply to college 1 or college 2 in DCA. Recall that the symmetric Bayesian Nash equilibrium characterized in Theorem 1 has the

Table 6: Observed effort choices and equilibrium predictions

Variable	Coefficient (Std. Err.)
Equilibrium effort	0.741*** (0.047)
Equilibrium effort in CCA	0.012 (0.083)
Dummy for CCA	-47.073 (30.451)
Intercept	194.628*** (17.922)
N	
	2160
R ²	
	0.306
F (4,11)	
	141.30

Notes: OLS estimation of effort levels based on clustered robust standard errors at the level of matching groups. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level.

property that students with an ability above the cutoff should always apply to the better college (college 2) whereas students with an ability below the cutoff should mix between the two colleges.

Result 5 (Choice of college in DCA): In DCA, students above the equilibrium ability cutoff choose the good college 2 more often than students below the cutoff. Across all markets and controlling for ability, the equilibrium predictions regarding the probability of choosing the good college have predictive power for the subjects' choices.

Support: Table 7 displays the cutoff ability for each market in the first column. In the second column it provides the average equilibrium probability of choosing the good college 2 for students with abilities below the cutoff in the respective markets. The average is calculated given the actual realization of abilities in the experiment. This can be compared to the observed frequency of choosing the good college in the experiment by these students in the next column. It emerges that subjects choose the good college 2 more often than predicted in all five markets, but in some markets the predicted and observed proportions are quite close. The next column (4) displays the proportion of subjects above the cutoff applying to college 2. Remember that in equilibrium these types should apply to college 2 with certainty. Finally, the last column of Table 7 presents the p-values for the test of equality of the proportions of the choice of college 2 below and above the market-specific equilibrium cutoff. In all markets the differences are significant at a 1% significance level.

Further evidence for the predictive power of the model is provided by Table 8. It shows the results of a probit model for the choice of the good college 2 in DCA. The coefficient for the

Table 7: Proportion of choices of good college 2

	Equilibrium ability cutoff (1)	Equ. prop. of choices of college 2 below the cutoff (2)	Obs. prop. of choices of college 2 below the cutoff (3)	Obs. prop. of choices of college 2 above the cutoff (4)	p-values for equality of proportions above and below the cutoff (5)
Market 1	50	13%	33%	85%	0.00
Market 2	85.5	43%	51%	92%	0.00
Market 3	85.5	15%	27%	68%	0.00
Market 4	89.5	16%	17%	42%	0.00
Market 5	23.5	51%	64%	91%	0.00

equilibrium probability of choosing the good college is significant at the 1% significance level.

Table 8: Choice of the good college 2 in DCA

Variable	Coefficient (Std. Err.)
Equilibrium probability of choosing the good college	1.684*** (0.106)
Intercept	-0.79*** (0.079)
N	1080
Pseudo R ²	0.177

Notes: Probit estimation of dummy for the choice of the good college based on clustered robust standard errors at the subject level. *** denotes statistical significance at the 1%-level, ** at the 5%-level, and * at the 10%-level.

Finally, we investigate the application decision of students by ability. Figure 3 presents the choices of subjects in DCA by markets and ability quantiles, together with the equilibrium proportions. Students above the equilibrium cutoff in market 1, market 2, and market 5 choose the good college 2 almost certainly, in line with the theory. The proportions of choices of students with low ability are also close to the equilibrium mixing probabilities. The biggest difference between the observed and the equilibrium proportions originates from the students who are slightly below the cutoff. This finding is particularly evident in markets 1, 2, and 4. To understand this, remember that the equilibrium is characterized by a discontinuity of the probability of the choice of college 2: students with abilities just above the cutoff have a pure strategy of choosing college 2, while students just below the cutoff choose college 1 with an almost 100% probability. Not surprisingly, in the experiment the choices of universities are smooth around the cutoff. Accordingly, we do not observe the predicted kink in the effort choices as is evident in Figure 2. This can be due to the fact that students with an ability level around the cutoff under- or overestimate the cutoff, which would result in the observed smoothing.

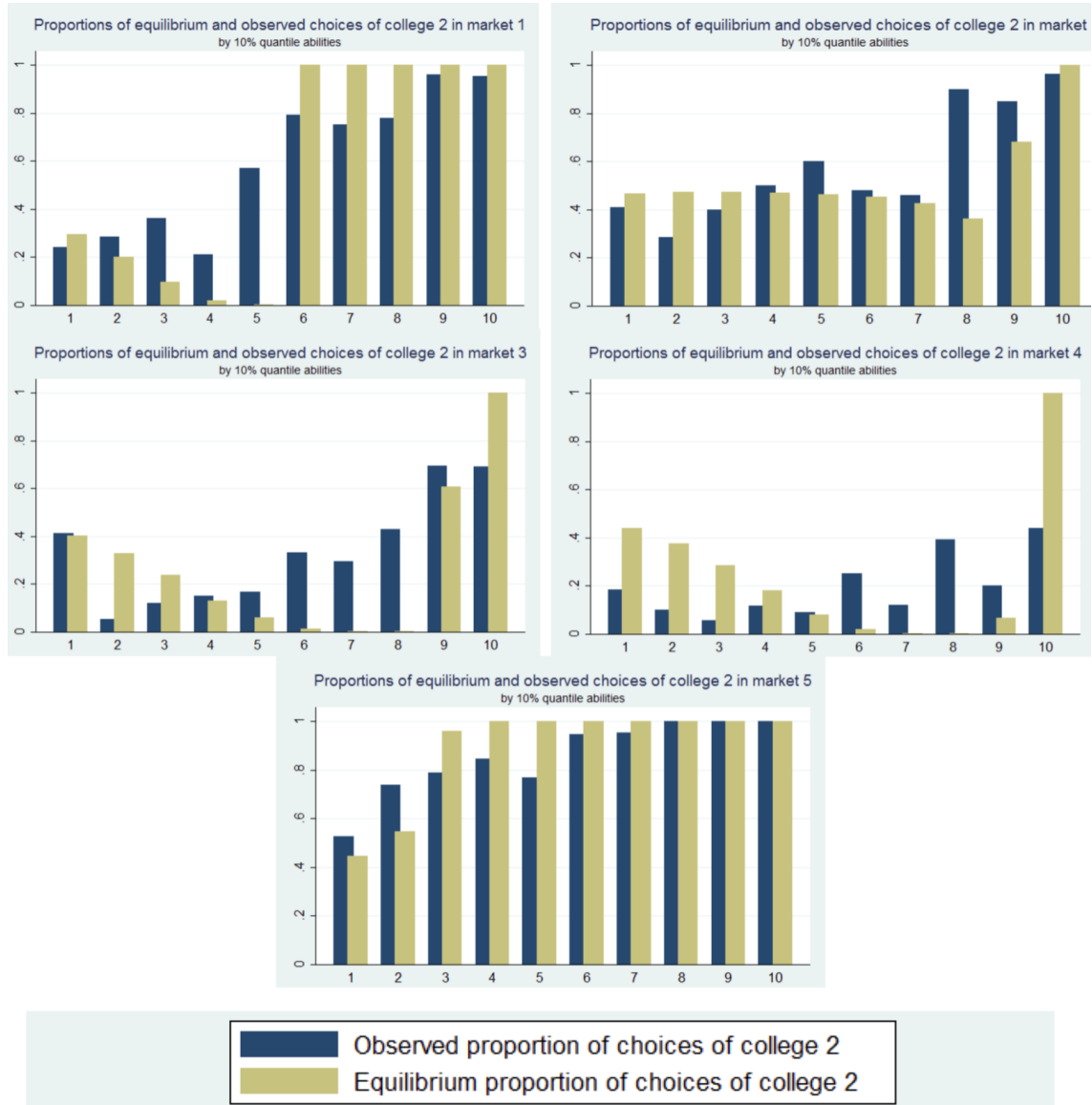


Figure 3: Choice of colleges by subjects in DCA

8 Conclusion

In this paper, we study college admissions exams which concern millions of students every year throughout the world. Our model abstracts away from many aspects of real-world college admission games and focuses on the following two important aspects: (i) colleges accept students by considering student exam scores, (ii) students have differing abilities which are their private information, and the costs of getting ready for the exams are inversely related to ability levels. Motivated by the Turkish and the Japanese college admissions mechanisms, we focus on two extreme policies. In the centralized model students can freely and costlessly apply to all colleges whereas in the decentralized mechanism, students can only apply to one college. We consider a model that is as simple as possible by assuming two colleges and homogeneous student preferences over colleges in order to derive analytical results as Bayesian Nash solutions to the two mechanisms.¹⁶

The solution of the centralized admissions mechanism follows from standard techniques in the contest literature. The solution to the decentralized model, on the other hand, has interesting properties such as lower ability students using a mixed strategy when deciding which school to apply to. Our main result is that low- and high-ability students differ in terms of their preferences between the two mechanisms where high-ability students prefer the centralized mechanism and low-ability students the decentralized mechanism.

We employ experiments to test the theory and to develop insights into the functioning of centralized and decentralized mechanisms that take into account behavioral aspects. We have implemented five different markets characterized by the common values of the two colleges to the students as well as the capacity of the two colleges. Overall, the main predictions of the theory are supported by the data, in spite of a few important differences. We find that in markets with an equal number of seats and applicants, the centralized mechanism is better for all applicants, as predicted by the theory. Again in line with the theory we observe that in markets with an overdemand for seats, low-ability students prefer a decentralized admissions mechanism whereas high-ability students prefer a centralized mechanism. However, in these markets we cannot confirm the predicted superiority of the decentralized mechanism for the students. This can be ascribed to one robust and stark difference between theory and observed behavior, namely overexertion of effort, which is more pronounced in the decentralized mechanism.

For the evaluation of the two mechanisms from a welfare perspective, it matters whether the effort spent preparing for the exam has no benefits beyond improving the performance in the exam or whether this effort is useful. If effort is purely a cost, then welfare can be measured by the mean utility of the students. In all our markets, the centralized mechanism at least weakly outperforms the decentralized mechanism with respect to this criterion. If the effort exerted by the students increases their productivity, then the decentralized mechanism becomes relatively more attractive, where efforts are weakly higher than in the centralized mechanism.

¹⁶We also discuss the extension to more colleges in section 6.

A Appendix

A.1 Preliminaries

The following lemmata are useful for the results given in the rest of the Appendix.

Lemma 1. *Let l, m be given integers. Then,*

$$\frac{d}{dx} \left(\sum_{j=0}^l p_{j,m-j}(x) \right) = -m p_{l,m-l-1}(x) \quad \text{when } 0 \leq l < m,$$

$$\frac{d}{dx} \left(\sum_{j=l}^m p_{j,m-j}(x) \right) = m p_{l-1,m-l}(x) \quad \text{when } 0 < l \leq m,$$

$$\frac{d}{dx} \left(\sum_{j=0}^l p_{m-j,j}(x) \right) = m p_{m-l-1,l}(x) \quad \text{when } 0 \leq l < m,$$

$$\frac{d}{dx} \left(\sum_{j=l}^m p_{m-j,j}(x) \right) = -m p_{m-l,l-1}(x) \quad \text{when } 0 < l \leq m.$$

Proof. We use the following equation:

$$\binom{m}{j-1} (m-j+1) = \frac{m!}{(j-1)!(m-j+1)!} (m-j+1) = \frac{m!}{(j-1)!(m-j)!} = \binom{m}{j} j. \quad (10)$$

The first formula: Suppose $0 = l$. Then, $\sum_{j=0}^l p_{j,m-j}(x) = p_{0,m}(x) = (1-x)^m$. Its derivative is $-m(1-x)^{m-1} = -m p_{0,m-1}(x)$. Thus the formula holds. Consider another case where $0 < l$. Then we have

$$\begin{aligned} \frac{d}{dx} \left(\sum_{j=0}^l p_{j,m-j}(x) \right) &= \frac{d}{dx} \left(\sum_{j=0}^l \binom{m}{j} x^j (1-x)^{m-j} \right) \\ &= \sum_{j=1}^l \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=0}^l \binom{m}{j} (m-j) x^j (1-x)^{m-j-1} \\ &= \sum_{j=1}^l \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=1}^{l+1} \binom{m}{j-1} (m-j+1) x^{j-1} (1-x)^{m-j} \\ &= \sum_{j=1}^l \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=1}^{l+1} \binom{m}{j} j x^{j-1} (1-x)^{m-j} \quad (\text{by (10)}) \end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d}{dx} \left(\sum_{j=0}^l p_{j,m-j}(x) \right) &= - \binom{m}{l+1} (l+1) x^l (1-x)^{m-l-1} = - \frac{m!}{l!(m-l-1)!} x^l (1-x)^{m-l-1} \\
&= - m \frac{(m-1)!}{l!(m-l-1)!} x^l (1-x)^{m-l-1} = -m p_{l,m-l-1}(x).
\end{aligned}$$

The second formula: Suppose $l = m$. Then, $\sum_{j=l}^m p_{j,m-j}(x) = p_{m,0}(x) = x^m$. Its derivative is $m x^{m-1} = m p_{m-1,0}(x)$. Thus the formula holds. Consider another case where $l < m$. Then we have

$$\begin{aligned}
\frac{d}{dx} \left(\sum_{j=l}^m p_{j,m-j}(x) \right) &= \frac{d}{dx} \left(\sum_{j=l}^m \binom{m}{j} x^j (1-x)^{m-j} \right) \\
&= \sum_{j=l}^m \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=l}^{m-1} \binom{m}{j} (m-j) x^j (1-x)^{m-j-1} \\
&= \sum_{j=l}^m \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=l+1}^m \binom{m}{j-1} (m-j+1) x^{j-1} (1-x)^{m-j} \\
&= \sum_{j=l}^m \binom{m}{j} j x^{j-1} (1-x)^{m-j} - \sum_{j=l+1}^m \binom{m}{j} j x^{j-1} (1-x)^{m-j} \quad (\text{by (10)})
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{d}{dx} \left(\sum_{j=0}^l p_{j,m-j}(x) \right) &= \binom{m}{l} l x^{l-1} (1-x)^{m-l} = \frac{m!}{(l-1)!(m-l)!} x^{l-1} (1-x)^{m-l} \\
&= m \frac{(m-1)!}{(l-1)!(m-l)!} x^{l-1} (1-x)^{m-l} = m p_{l-1,m-l}(x).
\end{aligned}$$

The third formula: By the second formula, we have

$$\frac{d}{dx} \left(\sum_{j=0}^l p_{m-j,j}(x) \right) = \frac{d}{dx} \left(\sum_{j=m-l}^m p_{j,m-j}(x) \right) = m p_{m-l-1,l}(x).$$

The fourth formula: By the first formula, we have

$$\frac{d}{dx} \left(\sum_{j=l}^m p_{m-j,j}(x) \right) = \frac{d}{dx} \left(\sum_{j=0}^{m-l} p_{j,m-j}(x) \right) = m p_{m-l,l-1}(x).$$

□

B On Theorem 1

B.1 Derivation of the symmetric equilibrium

We show how to obtain the function $\gamma : [0, c] \rightarrow (0, 1)$ and the cutoff c from Equation (6).

Step 1: We show that there is a unique value $\pi(c)$ that satisfies Equation (7). Define a function $\varphi_1 : [0, 1] \rightarrow \mathbb{R}$: for each $x \in [0, 1]$,

$$\varphi_1(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(x) - v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(x).$$

Differentiate φ_1 at each $x \in (0, 1)$: using Lemma 1, we have

$$\varphi_1'(x) = v_2(n-1)p_{(n-1)-(q_2-1)-1,q_2-1}(x) + v_1(n-1)p_{q_1-1,(n-1)-(q_1-1)-1}(x) > 0.$$

Thus, φ_1 is strictly increasing. Moreover, $\varphi_1(0) = -v_1 < 0$ and $\varphi_1(1) = v_2 > 0$. Thus, since φ_1 is a continuous function on $[0, 1]$, there is a unique $x^* \in (0, 1)$ such that $\varphi_1(x^*) = 0$. Thus, since $\varphi_1(\pi(c)) = 0$ by (7), there is a unique $\pi(c) \in (0, 1)$ that satisfies Equation (7).

Step 2: Given a unique $\pi(c)$, we now show that there is a unique cutoff $c \in (0, 1)$. In Equation (8), since $\pi(c)$ is known by Step 1, the the only unknown is c via $F(c)$. Define a function $\varphi_2 : [\pi(c), 1] \rightarrow \mathbb{R}$ as follows: for each $x \in [\pi(c), 1]$,

$$\varphi_2(x) = v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j} \left(\frac{x - \pi(c)}{1 - \pi(c)} \right) - v_1.$$

Differentiate φ_2 at each point $x \in (\pi(c), 1)$: using Lemma 1, we have

$$\varphi_2'(x) = v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \left(\frac{1}{1 - \pi(c)} \right) m p_{m-q_2,q_2-1} \left(\frac{x - \pi(c)}{1 - \pi(c)} \right) > 0.$$

Thus, φ is strictly increasing. Moreover, $\varphi_2(1) = v_2 - v_1 > 0$ and

$$\begin{aligned}
\varphi_2(\pi(c)) &= v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) + v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j}(0) - v_1 \\
&= v_2 \sum_{m=0}^{q_2-1} p_{n-m-1,m}(\pi(c)) - v_1 \quad (\because p_{j,m-j}(0) = 0 \text{ for } j \geq m - q_2 + 1 \geq 1) \\
&= v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi(c)) - v_1 \quad (\because (7)) \\
&< 0.
\end{aligned}$$

Therefore, there is a unique $x^* \in (\pi(c), 1)$ such that $\varphi_2(x^*) = 0$. Since $\varphi_2(F(c)) = 0$, $x^* = F(c)$. Thus, since F is strictly increasing, there is a unique cutoff $c \in (F^{-1}(\pi(c)), 1)$ such that $c = F^{-1}(x^*)$.

Step 3: From steps 1 and 2, $\pi(c)$ and c are uniquely determined. We now show that for each $a \in [0, c)$, there is a unique $\pi(a) \in (0, 1)$ that satisfies (9). Fix $a \in [0, c)$. Define a function $\varphi_3 : [0, F(a)] \rightarrow \mathbb{R}$:

$$\varphi_3(x) = v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j}\left(\frac{F(a)-x}{1-\pi(c)}\right) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j}\left(\frac{x}{\pi(c)}\right).$$

Let us differentiate φ_3 at each $x \in (0, F(a))$ by using Lemma 1:

$$\begin{aligned}
\varphi_3'(x) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \left(-\frac{1}{1-\pi(c)}\right) m p_{m-q_2,q_2-1}\left(\frac{F(a)-x}{1-\pi(c)}\right) \\
&\quad - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \left(\frac{1}{\pi(c)}\right) m p_{m-q_1,q_1-1}\left(\frac{x}{\pi(c)}\right) < 0.
\end{aligned}$$

Thus, φ is strictly decreasing. Moreover,

$$\begin{aligned}
\varphi_3(0) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j}\left(\frac{F(a)}{1-\pi(c)}\right) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j}(0) \\
&= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j}\left(\frac{F(a)}{1-\pi(c)}\right) \quad (\because p_{j,m-j}(0) = 0) \\
&> 0.
\end{aligned}$$

and

$$\begin{aligned}
\varphi_3(F(a)) &= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \sum_{j=m-q_2+1}^m p_{j,m-j}(0) - v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j}\left(\frac{F(a)}{\pi(c)}\right) \\
&= -v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \sum_{j=m-q_1+1}^m p_{j,m-j}\left(\frac{F(a)}{\pi(c)}\right) \quad (\because p_{j,m-j}(0) = 0) \\
&< 0.
\end{aligned}$$

Thus, there is a unique $x^* \in (0, F(a))$ such that $\varphi_3(x^*) = 0$. Since $\varphi_3(\pi(a)) = 0$, $x^* = \pi(a)$. Hence, there is a unique $\pi(a) \in (0, 1)$ that satisfies Equation (9).

Step 4: Finally, we derive $\gamma(a)$ for each $a \in (0, c)$. Recall that in (9), $\pi(a) = \int_0^a \gamma(x)f(x)dx$ and $\pi(c)$ and $\pi(a)$ are known by previous steps. Differentiate (9) with respect to a by using Lemma 1:

$$\begin{aligned}
&v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) \left(\frac{\gamma(a)f(a)}{\pi(c)}\right) m p_{m-q_1,q_1-1}\left(\frac{\pi(a)}{\pi(c)}\right) \\
&= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) \left(\frac{f(a) - \gamma(a)f(a)}{1 - \pi(c)}\right) m p_{m-q_2,q_2-1}\left(\frac{F(a) - \pi(a)}{1 - \pi(c)}\right). \quad (11)
\end{aligned}$$

Let us define the following functions:

$$\begin{aligned}
A(a) &:= v_1 \sum_{m=q_1}^{n-1} p_{m,n-m-1}(\pi(c)) m p_{m-q_1,q_1-1}\left(\frac{\pi(a)}{\pi(c)}\right) > 0, \\
B(a) &:= v_2 \sum_{m=q_2}^{n-1} p_{n-m-1,m}(\pi(c)) m p_{m-q_2,q_2-1}\left(\frac{F(a) - \pi(a)}{1 - \pi(c)}\right) > 0.
\end{aligned}$$

Then, we can write (11) as

$$\frac{\gamma(a)f(a)}{\pi(c)} A(a) = \frac{f(a)(1 - \gamma(a))}{1 - \pi(c)} B(a). \quad (12)$$

Solving for $\gamma(a)$ in (12), we obtain

$$\gamma(a) = \frac{\pi(c)B(a)}{(1 - \pi(c))A(a) + \pi(c)B(a)} \in (0, 1).$$

By construction, function γ we have derived satisfies Equation (9).

B.2 Verification: the candidate is an equilibrium

In this appendix, we check for global deviations and confirm that the unique symmetric equilibrium candidate we have derived in Theorem 1 is indeed an equilibrium. As a preliminary notation and analysis, let us calculate the probability, denoted by $P[1, b|c, \gamma, \beta^D]$, that a student who makes

effort $e = \beta^D(b)$ and applies to college 1 ends up getting a seat in college 1:

$$P[1, b|\gamma, \beta^D] = \begin{cases} \sum_{m=0}^{q_1-1} \hat{p}_{m, n-m-1}(c) + \sum_{m=q_1}^{n-1} \hat{p}_{m, n-m-1}(c) G_{m-q_1+1, m}(b) & \text{if } b \in [0, c] \\ 1 & \text{if } b \geq c. \end{cases}$$

Obviously, if the student chooses an effort more than $\beta(c)$, he will definitely get a seat in college 1. Otherwise, the first line represents the sums of the probability of events in which e is one of the highest q_1 efforts among the students who apply to college 1.

Similarly, let us calculate the probability, denoted by $P[2, b|\beta, \gamma]$, that a student who makes effort $e = \beta(b)$ and applies to college 2 ends up getting a seat in college 2.

$$P[2, b|\gamma, \beta^D] = \begin{cases} \sum_{m=0}^{q_2-1} \hat{p}_{n-m-1, m}(c) + \sum_{m=q_2}^{n-1} \hat{p}_{n-m-1, m}(c) H_{m-q_2+1, m}(b) & \text{if } b \in [0, 1] \\ 1 & \text{if } b \geq 1. \end{cases}$$

Obviously, if the student chooses an effort more than $\beta(1)$, he will definitely get a seat in college 2.¹⁷ Otherwise, the first line represents the sums of probability of events in which e is one of the highest q_2 efforts among the students who apply to college 2.

Next, denote by $U(r, b|\gamma, \beta^D, a)$ (or $U(r, b|a)$ for short) the expected utility of type a who chooses college 1 with probability r and makes effort $e = \beta^D(b)$ when all of the other students follow the strategy (γ, β^D) . We have,

$$U(r, b|a) := rP[1, b|\gamma, \beta^D]v_1 + (1-r)P[2, b|\gamma, \beta^D]v_2 - \frac{e}{a}.$$

We need to show that for each $a \in [0, 1]$, each $r \in [0, 1]$ and each $b \geq 0$, $\hat{U}(a) \equiv U(\gamma(a), a|a) \geq U(r, b|a)$. Fix $a \in [0, 1]$. It is sufficient to show that $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$, as these two conditions together implies required “no global deviation” condition. Below, we show that for any $a \in [0, 1]$, and for $b \geq 0$, both $\hat{U}(a) \geq U(0, b|a)$ and $\hat{U}(a) \geq U(1, b|a)$ hold. We consider two cases, one for lower ability students ($a \in [0, c]$), one for higher ability students ($a \in [c, 1]$). As sub-cases, we analyze b to be in the same region (b is low for a low, and b is high for a high), different region (a high, b low; and a low, b high), and b being over 1. The no-deviation results for the same region is standard, whereas deviations across regions need to be carefully analyzed.

Case 1: Type $a \in [0, c]$

Case 1-1: $b \in [0, c]$. Then, by our derivation, we have $U(0, b|a) = U(1, b|a)$ and also $\hat{U}(a) \geq U(1, b|a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Hence, we can conclude that $\hat{U}(a) \geq U(1, e|a) = U(0, e|a)$.

Case 1-2: $b \in (c, 1]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

¹⁷Of course, there is no type b with $b > 1$, if a student chooses an effort e strictly greater than $\beta^D(1)$, we represent him as mimicking a type $b > 1$.

$$\begin{aligned}
\hat{U}(a) &\geq U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{\beta^D(b)}{a} \quad (\because \beta^D(c) \leq \beta^D(b)). \\
&= U(1, b|a).
\end{aligned}$$

Next, we show $\hat{U}(a) \geq U(0, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = P[2, c|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{a} \\
&= \left(P[2, \beta^D(c)|\gamma, \beta^D]v_2 - \frac{\beta^D(c)}{c} \right) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} = U(0, c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(0, b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} = P[2, b|\gamma, \beta^D]v_2 - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= \left(P[2, b|\gamma, \beta^D] - \frac{\beta^D(b)}{a} \right) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(0, b|a) + (\beta^D(b) - \beta^D(c)) \left(\frac{1}{a} - \frac{1}{c} \right) \\
&\geq U(0, b|a) \quad (\because \beta^D(b) \geq \beta^D(c), a < c).
\end{aligned}$$

Case 1-3: $b > 1$ (or $e > \beta^D(1)$).

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&> v_1 - \frac{e}{a} \quad (\because \beta^D(c) \leq \beta^D(1) < e) \\
&= U(1, b|a).
\end{aligned}$$

Moreover,

$$\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) \quad (\text{by Case 1-2}) \\
&= v_2 - \frac{\beta^D(1)}{a} \\
&> v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}$$

Case 2: Type $a \in [c, 1]$

Case 2-1: $b \in [0, c]$. We first show $\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= U(\gamma(c), c|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&\geq U(\gamma(b), b|c) + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(\gamma(b), b|a) + \frac{\beta^D(b)}{a} - \frac{\beta^D(b)}{c} + \frac{\beta^D(c)}{c} - \frac{\beta^D(c)}{a} \\
&= U(1, b|a) + (\beta^D(c) - \beta^D(b)) \left(\frac{1}{c} - \frac{1}{a} \right) \quad (\because U(\gamma(b), b|a) = U(1, b|a)) \\
&\geq U(1, b|a) \quad (\because \beta^D(c) - \beta^D(b) \geq 0, c < a).
\end{aligned}$$

To obtain $\hat{U}(a) \geq U(0, b|a)$, note that in the above inequalities, if we use $U(\gamma(b), b|a) = U(0, b|a)$ in the fourth line, we obtained the desired inequality.

Case 2-2: $b \in (c, 1]$. First, by our derivation, $\hat{U}(a) \geq U(0, e|\gamma, \beta^D, a)$ can be shown via standard arguments (for instance, see section 3.2.1 and Proposition 2.2 in Krishna, 2002). Next, we show $\hat{U}(a) \geq U(1, b|a)$.

$$\begin{aligned}
\hat{U}(a) &\geq U(0, c|a) = v_2 P[2, c|\gamma, \beta^D] - \frac{\beta^D(c)}{a} \\
&= v_1 - \frac{\beta^D(c)}{a} \quad (\because v_2 P[2, c|\gamma, \beta^D] = v_1) \\
&\geq v_1 - \frac{\beta^D(b)}{a} = U(1, b|a) \quad (\because \beta^D(c) \leq \beta^D(b)).
\end{aligned}$$

Case 2-3: $b > 1$ (or $e > \beta^D(1)$)

$$\begin{aligned}
\hat{U}(a) &\geq U(\gamma(c), c|a) = U(1, c|a) = v_1 - \frac{\beta^D(c)}{a} \\
&\geq v_1 - \frac{e}{a} \quad (\because e > \beta^D(1) > \beta^D(c)) \\
&\geq U(1, b|a).
\end{aligned}$$

and

$$\begin{aligned}
\hat{U}(a) &\geq U(0, 1|a) = v_2 - \frac{\beta^D(1)}{a} \\
&\geq v_2 - \frac{e}{a} \quad (\because e > \beta^D(1)) \\
&= U(0, b|a).
\end{aligned}$$

C Equilibrium Derivation for l Colleges

We show how to derive cutoffs, mixed strategies, and cost functions provided there exists an equilibrium as specified in section 6. The basic procedure follows the one in Theorem 1.

We first show how to obtain the equilibrium cutoffs c_1, \dots, c_{l-1} and the mixed strategy function $\gamma_1, \dots, \gamma_{l-1}$. Let $k \in \{1, \dots, l-1\}$. A necessary condition for this to be an equilibrium is that each type $a \in [c_{k-1}, c_k]$ has to be indifferent between applying to college 1 and college 2. Thus, for all $a \in [c_{k-1}, c_k]$,

$$\begin{aligned} & v_k \left(\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) H_{m-q_k+1,m}^k(a) \right) \\ &= v_{k+1} \left(\sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) + \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) H_{m-q_{k+1}+1,m}^{k+1}(a) \right). \end{aligned} \quad (13)$$

Step 1: Find $\pi^1(c_1), \dots, \pi^l(c_l)$. Equation (13) can be written as

$$\begin{aligned} v_1 \sum_{m=0}^{q_1-1} p_{m,n-m-1}(\pi^1(c_1)) &= v_2 \sum_{m=0}^{q_2-1} p_{m,n-m-1}(\pi^2(c_2)), \\ v_{k-2} &= v_k \sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) \quad \text{for } k \in \{3, \dots, l\}, \end{aligned} \quad (14)$$

where the first equation is Equation (13) at $a = 0$ under $k = 1$, which says that a type $a = 0$ is indifferent between college 1 and 2; the second equation follows from Equation (13) at $a = c_k$ under $k-1$ and k , which says that a type $a = c_{k-2}$ is indifferent between colleges $k-2$ and k . Therefore, $\pi^1(c_1), \dots, \pi^l(c_l)$ can be obtained by solving Equation (14).

Step 2: Given $\pi^1(c_1), \dots, \pi^l(c_l)$, find cutoffs c_1, \dots, c_{l-1} . We first show the following claim that shows how to obtain $\pi^k(c_{k-1})$ from $\pi^1(c_1), \dots, \pi^l(c_l)$. $\pi^k(c_{k-1}) = F(c_{k-1}) - \sum_{j=1}^{k-1} \pi^j(c_j)$.

Proof. For $k = 2$: Note that $\pi^1(c_1) = \int_0^{c_1} \gamma_1(x) dF(x)$. Thus $\pi^2(c_1) := \int_0^{c_1} (1 - \gamma_1(x)) dF(x) = F(c_1) - \pi^1(c_1)$. Suppose that the claim is true up to $k-1$ where $k \geq 3$. Then $\pi^{k-1}(c_{k-1}) := \pi^{k-1}(c_{k-2}) + \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x) dF(x)$. Thus $\int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x) dF(x) = \pi^{k-1}(c_{k-1}) - \pi^{k-1}(c_{k-2})$. Hence, by

the induction hypothesis, we have

$$\begin{aligned}
\pi^k(c_{k-1}) &:= \int_{c_{k-2}}^{c_{k-1}} (1 - \gamma_{k-1}(x)) dF(x) \\
&= F(c_{k-1}) - F(c_{k-2}) - \int_{c_{k-2}}^{c_{k-1}} \gamma_{k-1}(x) dF(x) \\
&= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + \pi^{k-1}(c_{k-2}) \\
&= F(c_{k-1}) - F(c_{k-2}) - \pi^{k-1}(c_{k-1}) + (F(c_{k-2}) - \sum_{j=1}^{k-2} \pi^j(c_j)) \\
&= F(c_{k-1}) - \sum_{j=1}^{k-1} \pi^j(c_j).
\end{aligned}$$

□

Now Equation (13) at $a = c_k$ can be rewritten as, for each $k \in \{1, \dots, l-1\}$,

$$\begin{aligned}
v_k &= v_{k+1} \sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\
&\quad + v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \sum_{j=m-q_{k+1}+1}^m p_{j,m-j} \left(\frac{F(c_k) - (\pi^1(c_1) + \dots + \pi^k(c_k))}{\pi^{k+1}(c_{k+1})} \right), \quad (15)
\end{aligned}$$

where we use induction claim and

$$H_{m-q_{k+1}+1, m}^{k+1}(c_k) = \sum_{j=m-q_{k+1}+1}^m p_{j,m-j} \left(\frac{\pi^{k+1}(c_k)}{\pi^{k+1}(c_{k+1})} \right).$$

Hence, given $\pi^1(c_1), \dots, \pi^l(c_l)$, we can find c_k by solving Equation (15).

Step 3: Given $\pi^1(c_1), \dots, \pi^l(c_l)$ and c_1, \dots, c_{l-1} , for each $k \in \{1, \dots, l-1\}$ and each $a \in [c_{k-1}, c_k]$, there is a unique $\pi^k(a)$ that satisfies Equation (16). Moreover, we can get the mixed strategy function $\gamma^k(a)$ by differentiating Equation (16).

Equation (13) at $a \in [c_{k-1}, c_k]$ can be rewritten as, for each $k \in \{1, \dots, n-1\}$,

$$\begin{aligned}
v_k &\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) \sum_{j=m-q_k+1}^m p_{j,m-j} \left(\frac{\pi^k(a)}{\pi^k(c_k)} \right) \\
&= v_{k+1} \sum_{m=0}^{q_{k+1}-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \\
&\quad + v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \sum_{j=m-q_{k+1}+1}^m p_{j,m-j} \left(\frac{F(a) - F(c_{k-1}) - \pi^k(a) + \pi^k(c_{k-1})}{\pi^{k+1}(c_{k+1})} \right) \quad (16)
\end{aligned}$$

where we used the following equation: for each $a \in [c_{k-1}, c_k]$, since $\pi^k(a) := \pi^k(c_{k-1}) + \int_{c_{k-1}}^a \gamma_k(x) dF(x)$,

$$\begin{aligned}\pi^{k+1}(a) &:= \int_{c_{k-1}}^a (1 - \gamma_k(x)) dF(x) \\ &= F(a) - F(c_{k-1}) - \pi^k(a) + \pi^k(c_{k-1}).\end{aligned}$$

Differentiate Equation (16) with respect to a by using Lemma 1:

$$\begin{aligned}& v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) \frac{\gamma_k(a)f(a)}{\pi^k(c_k)} m p_{m-q_k, q_k-1} \left(\frac{\pi^k(a)}{\pi^k(c_k)} \right) \\ &= v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) \frac{f(a) - \gamma_k(a)f(a)}{\pi^{k+1}(c_{k+1})} m p_{m-q_{k+1}, q_{k+1}-1} \left(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})} \right).\end{aligned}\quad (17)$$

Let us define the following functions:

$$\begin{aligned}A^k(a) &= v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) m p_{m-q_k, q_k-1} \left(\frac{\pi^k(a)}{\pi^k(c_k)} \right) > 0 \\ B^k(a) &= v_{k+1} \sum_{m=q_{k+1}}^{n-1} p_{m,n-m-1}(\pi^{k+1}(c_{k+1})) m p_{m-q_{k+1}, q_{k+1}-1} \left(\frac{\pi^{k+1}(a)}{\pi^{k+1}(c_{k+1})} \right) > 0.\end{aligned}$$

Then we can write (17) as

$$\frac{\gamma_k(a)f(a)}{\pi^k(c_k)} A^k(a) = \frac{f(a)(1 - \gamma_k(a))}{\pi^{k+1}(c_{k+1})} B^k(a).\quad (18)$$

Solving for $\gamma_k(a)$ in (18), we obtain

$$\gamma_k(a) = \frac{\pi^k(c_k) B^k(a)}{\pi^{k+1}(c_{k+1}) A^k(a) + \pi^k(c_k) B^k(a)}.$$

Step 4: We find the effort function β^D . Consider a student with type $a \in [c_{k-1}, c_k]$. A necessary condition is that she does not want to mimic any other type a' in $[c_{k-1}, c_k]$. Her utility maximization problem is

$$\max_{a' \in [c_{k-1}, c_k]} v_k \left(\sum_{m=0}^{q_k-1} p_{m,n-m-1}(\pi^k(c_k)) + \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) H_{m-q_k+1, m}^k(a') \right) - \frac{\beta^D(a')}{a}.$$

The first-order necessary condition requires the derivative of the objective function to be 0 at $a' = a$. Hence

$$v_k \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h_{m-q_k+1,m}(a) - \frac{(\beta^D(a))'}{a} = 0.$$

Solving the differential equation with the boundary condition at $\beta^D(c_{k-1})$, we obtain

$$\beta^D(a) = \beta^D(c_{k-1}) + v_k \int_{c_{k-1}}^a x \sum_{m=q_k}^{n-1} p_{m,n-m-1}(\pi^k(c_k)) h_{m-q_k+1,m}^k(x) dx$$

for all $a \in [c_{k-1}, c_k]$.

D Experiment Instruction(not for publication)

Welcome! This is an experiment about decision making. You and the other participants in the experiment will participate in a situation where you have to make a number of choices. In this situation, you can earn money that will be paid out to you in cash at the end of the experiment. How much you will earn depends on the decisions that you and the other participants in the experiment make.

During the experiment you are not allowed to use any electronic devices or to communicate with other participants. Please use exclusively the programs and functions that are intended to be used in the experiment.

These instructions describe the situation in which you have to make a decision. The instructions are identical for all participants in the experiment. It is important that you read the instructions carefully so that you understand the decision-making problem well. If something is unclear to you while reading, or if you have other questions, please let us know by raising your hand. We will then answer your questions individually.

Please do not, under any circumstances, ask your question(s) aloud. You are not permitted to give information of any kind to the other participants. You are also not permitted to speak to other participants at any time throughout the experiment. Whenever you have a question, please raise your hand and we will come to you and answer it. If you break these rules, we may have to terminate the experiment.

Once everyone has read the instructions and there are no further questions, we will conduct a short quiz where each of you will complete some tasks on your own. We will walk around, look over your answers, and solve any remaining comprehension problems. The only purpose of the quiz is to ensure that you thoroughly understand the crucial details of the decision-making problem.

Your anonymity and the anonymity of the other participants will be guaranteed throughout the entire experiment. You will neither learn about the identity of the other participants, nor will they learn about your identity.

General description

This experiment is about students who try to enter the university. The 24 participants in the room are grouped into two groups of 12 persons each. These 12 participants represent students competing for university seats. The experiment consists of 15 independent decisions (15 rounds), which represent different student admission processes. At the end of each round every student will receive at most one seat in one of the universities or will remain unassigned.

There are two universities that differ in quality. We refer to the best university as University 1. Admission to the best university (University 1) yields a payoff of 2,000 points for the students. Admission to University 2 yields a smaller payoff for the students, which can vary across the rounds. Each university has a certain number of seats to be filled, a factor which can also be different for each of the rounds.

Instructions for CCA

The allocation procedure is implemented in the following way:

At the beginning of the each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

Admission to universities is centralized and is based on the amount of effort that each student puts into a final exam. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) the effort. The higher the ability the easier (cheaper) the effort. The price of one unit of effort is determined as: $100 / \text{ability}$. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen. You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding of the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can either be used to exert effort or kept.

After each student has decided how much effort to buy, these effort levels are sent to the centralized clearing house which then determines the assignments to universities. The students who have chosen the highest effort levels are assigned to University 1 up to the capacity of this university. They receive 2,000 points. The students with the next higher levels of effort are assigned to University 2 up to its capacity and receive the corresponding amount of points. All other students who have applied remain unassigned and will receive no points. Participants that

have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff from university admission. Thus:

$$\text{Payoff} = \text{Endowment} - \text{price of effort} * \text{units of effort} + \text{payoff from assignment}$$

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for your actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats. The admission to University 1 yields 2,000 points and the admission to University 2 yields 1,000 points.

Julia has an ability of 25. Thus the cost of one unit of effort is $100/25 = 4$ points for her. Her endowment is 2,200 points, which means that she can buy a maximum of $2,200/4 = 550$ units of effort. Let us imagine that Julia decided to buy 400 units of effort. Thus she has to pay $400 * 4 = 1,600$ points and keeps 600 points of her endowment.

Peter has an ability of 50. Thus the cost of effort for him is $100/50 = 2$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2,200/2 = 1100$ units of effort. Let us assume that Peter chose 600 units of effort. Thus he has to pay $600 * 2 = 1,200$ points.

Simon has an ability of 80. Thus the cost of one unit of effort is $100/80 = 1.25$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2,200/1.25 = 1760$ units of effort. Let us imagine that Simon decides to buy 500 units of effort. Thus he has to pay $500 * 1.25 = 625$ points.

Imagine that the following effort levels were chosen by the other 9 participants: 10, 70, 200, 250, 420, 450, 550, 700, 1,200.

Thus, the four students with the highest effort levels are assigned to University 1 and receive a payoff of 2,000 points. These are the students with effort levels 1,200, 700, 600 (Peter), and 550. Of the remaining eight students, five students with the highest levels of efforts are assigned to University 2 and receive a payoff of 1,000 points. These are the students with the effort levels 500 (Simon), 450, 420, 400 (Julia) and 250.

The students with effort levels 10, 70, and 200 remain unassigned.

Thus, the payoff for Julia is $2,200 - 1,600 + 1,000 = 1,600$, for Peter $2,200 - 1,200 + 2,000 = 3,000$ and for Simon $2,200 - 625 + 1,000 = 2,575$.

Instructions for DCA

The allocation procedure is implemented as follows:

At the beginning of the each round, every student learns her ability. The ability of each student is drawn uniformly from the interval from 0 to 100. Thus every student has an equal chance of being assigned every level of ability from the interval. You will learn your own ability but not the ability of the other 11 students competing with you for the seats. The ability is drawn independently for all participants in every round.

The admission to universities is decentralized. Students first decide which university they want to apply to. Thus, you have to choose one university you want to apply to. After the decision is made, you will compete only with students who have decided to apply to the same university. The assignment of seats at each university is based on the amount of the effort that each student puts into a final test. In the experiment you can choose a level of effort. This effort is costly. The price of effort depends on your ability. The higher the ability the easier (cheaper) is the effort. The price of one unit of effort is determined as: 100 divided by the ability, $100/\text{ability}$. On your screen you will see your ability for the round and the corresponding price of one unit of effort. You will have to decide on the amount of the effort that you choose.

In each of the rounds you can use the calculator which will be on your screen. You can use it to find out what possible payoffs a particular effort in points can yield. To gain a better understanding for the experiment you can insert different values. This will help you with your decision.

In the beginning of each round, every participant receives 2,200 points that can be used to exert effort or kept.

After each student decides how much effort to buy, these efforts are used to determine the assignments to universities. Among the students who apply to University 1, the students with the highest effort levels are assigned to this university up to its capacity and receive 2,000 points. All other students who applied to University 1 remain unassigned. Among those students who apply to University 2, the students with the highest effort levels are assigned a seat up to the capacity of University 2. They receive the corresponding amount of points. All other students who have applied to University 2 remain unassigned. Participants that have chosen the same amount of effort will be ranked according to a random draw.

Each participant receives a payoff that is determined as the sum of the non-invested endowment and the payoff from university admission. Thus:

$$\text{Payoff} = \text{Endowment} - \text{price of effort} * \text{units of effort} + \text{payoff from assignment}$$

Note that your ability, the ability of the other participants, and the number of seats at University 1 and University 2 vary in every round.

Every point corresponds to 0.5 cents. Only one of the rounds will be relevant for you actual payoff. This round will be selected randomly by the computer at the end of the experiment.

Example

Let us consider an example with three hypothetical persons: Julia, Peter, and Simon.

Imagine the following round: University 1 has four seats, and University 2 has five seats.

Julia has an ability of 25 and decides to apply to University 2. Thus the cost of one unit of effort is $100/25 = 4$ points for her. Her endowment is 2,200 points, which means that she can buy a maximum of $2,200/4 = 550$ units of effort. Let us imagine that Julia decided to buy 400 units of effort. Thus she has to pay $400 \cdot 4 = 1,600$ points and keeps 600 points of her endowment.

Peter has an ability of 50. He applies to University 1. Thus the cost of effort for him is $100/50 = 2$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2,200/2 = 1100$ units of effort. Let us assume that Peter chose 600 units of effort. Thus he has to pay $600 \cdot 2 = 1,200$ points.

Simon has an ability of 80. He applies to University 2. Thus the cost of one unit of effort is $100/80 = 1.25$ points for one unit of effort. His endowment is 2,200 points. Thus he can buy a maximum of $2,200/1.25 = 1,760$ units of effort. Let us imagine that Simon decides to buy 500 units of effort. Thus he has to pay $500 \cdot 1.25 = 625$ points.

Imagine that there are an additional four students who decide to apply to University 2 (competing with Julia and Simon), and five students who decide to apply to University 1 (competing with Peter). The following efforts were bought by the four participants who apply to University 2, together with Julia: 10, 70, 450, 550.

Thus, there are 6 contenders for 5 seats. All students, but one with the effort of 10, receive a seat at University 2 and thus a payoff of 1,000 points.

The following efforts were bought by the five other participants who apply to University 1, together with Peter: 200, 250, 420, 700, 1,200.

Thus, there are 6 contenders for 4 seats. The four students with the highest efforts are assigned to University 1, including Peter, and all receive 2,000 points.

The students with effort levels 200 and 250 remain unassigned.

Thus, the payoff for Julia is $2,200 - 1,600 + 1,000 = 1,600$, for Simon $2,200 - 625 + 1,000 = 2,575$ and for Peter $2,200 - 1,200 + 2,000 = 3,000$.

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