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Sabotage in Dynamic Tournaments

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ABSTRACT

Sabotage in Dynamic Tournaments

by Oliver Gürtler and Johannes Münster *

This paper studies sabotage in a dynamic tournament. Three players compete in two rounds. In the final round, a player who is leading in the race, but not yet beyond the reach of his competitors, gets sabotaged more heavily. As a consequence, if players are at the same position initially, they do not work productively or sabotage at all in the first round. Thus sabotage is not only directly destructive, but also depresses incentives to work productively. If players are heterogeneous ex ante, sabotage activities in the first round may be concentrated against an underdog, contrary to findings from static tournaments. We also discuss the robustness of our results in a less stylized environment.

Keywords: Dynamic tournaments, contests, sabotage, heterogeneity

ZUSAMMENFASSUNG

Sabotage in dynamischen Turnieren

Dieser Aufsatz untersucht Sabotage in einem dynamischen Turnier. Drei Spieler konkurrieren in zwei Runden. In der letzten Runde wird ein Spieler, der einen Vorsprung hat, aber noch einholbar ist, stärker sabotiert. Deshalb wird eine Gruppe mit homogenen Spielern in der ersten Runde weder produktiv arbeiten noch sabotieren. Sabotage ist also nicht nur direkt destruktiv, sondern verringert auch die Anreize zu produktiver Anstrengung. Wenn die Spielergruppe ex-ante heterogen ist, kann sich Sabotage in der ersten Runde überwiegend gegen schwächere Spieler richten, im Gegensatz zu Ergebnissen aus statischen Turnieren. Unsere Ergebnisse bleiben auch in einem weniger eng definierten Modellrahmen robust bestehen.

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1 Introduction

In practice, tournaments or contests are ubiquitous. A typical example is an internal labor market tournament, in which employees compete for a bonus or a promotion (Lazear and Rosen 1981). Other examples include R&D races, litigation contests, rent seeking contests, political campaigning or sports contests.[1]

In tournaments, only a relative comparison of the contestants is important. The players therefore have an incentive to sabotage each other, and the consequences of sabotage in static tournaments are by now relatively well understood (see e.g. Lazear 1989, Drago and Turnbull 1991, Skaperdas and Grofman 1995, Chan 1996, Drago and Garvey 1998, Konrad 2000, Kräkel 2000, Chen 2003, 2005, Harbring, Irlenbusch, Kräkel, and Selten 2007, Münster 2007, Gürtler 2008). Many real world tournaments, however, take place over a certain time period, and thus are dynamic in nature. Dynamic tournaments have been studied in the literature on R&D races (e.g. Fudenberg, Gilbert, Stiglitz, and Tirole, 1983, Harris and Vickers 1985). Newer papers include Konrad and Kovenock (2006), Klumpp and Polborn (2006) on election races, and Yildirim (2005) on rent-seeking contests. Very little is known, however, about sabotage in dynamic tournaments.[2] An exception is Ishida (2006), who studies sabotage in a dynamic tournament between two players with asymmetric information about their abilities. Ishida points to a ratchet effect, where high ability contestants have an incentive to hide their ability early on, in order to avoid becoming a victim of sabotage, and focusses on the optimal design of sabotage-proof contracts which lead to zero sabotage in equilibrium.

Our paper is located at the intersection between the literature on sabo-

[1] See Konrad (2007) for a survey.

[2] An interaction between sabotage and dynamic considerations is analyzed in Auriol, Friebel, and Pechlivanos (2002), who show that career concerns may induce sabotage even if players are not competing in a tournament.

tage in tournaments and on dynamic tournaments and studies sabotage in a dynamic tournament. There are three players competing in two rounds for a prize. In contrast to Ishida (2006), we assume that players' abilities are common knowledge; thus there is no ratchet effect in our model. Initially, players have some exogenously given position. In each round, a player can work productively and move one step forward; moreover, he can sabotage one of his rivals and move him one step backward. Sabotage, however, is more costly than productive effort. Between the rounds, players observe each others' position. After the second round, the player who is leading wins; ties are broken randomly.

When contestants are heterogeneous, a common finding concerning static tournaments is that favorites are sabotaged more strongly than underdogs since they are the more dangerous rivals (Chen 2003, Harbring, Irlenbusch, Kräkel, and Selten 2007, Münster 2007). This is replicated in the final round of our model. We show that, as a consequence, sabotage may decrease incentives in the first round. In fact, when players are homogeneous ex ante, i.e. start at the same position, no player moves forward or sabotages any rival in the first round to avoid being sabotaged in round two. Thus sabotage is not only directly harmful by destroying valuable output, but also indirectly depresses the incentives to work productively.

Our second main result is on the question who is likely to be a victim of sabotage. In the first round, we show that sabotage may be concentrated against an underdog to basically eliminate him from the tournament. As explained above, this is very different from findings in static tournaments.

The remainder of the paper is organized as follows. Section 2 introduces a stylized model of sabotage in a two-stage tournament. Proceeding backwards, Sections 3 and 4 study equilibria of the second and the first stage, respectively. Section 5 discusses the robustness of the results obtained, by pointing out other equilibria (Section 5.1), discussing the assumptions driving the main result (Section 5.2), and by showing that a similar result can be obtained in a less stylized model (Section 5.3).

2 The model

Consider a two-round tournament game ($t = 1, 2$) with three players $i = 1, 2, 3$ competing for a prize $V > 0$. In each of the rounds, player i chooses a productive effort $e_i^t \in \{0, 1\}$ and a sabotage effort $s_{ij}^t \in \{0, 1\}$, $j = 1, 2, 3$, $j \neq i$. We assume $\sum_{j \neq i} s_{ij}^t \leq 1$, i.e. a player cannot sabotage both his opponents at the same time. To highlight the strategic effects of the model, we impose the following assumption.

Assumption 1 A productive effort of 1 entails no costs for a player. Player i chooses $e_i^t = 1$ unless this strictly reduces his payoff.

Sabotage, instead, leads to costs $k > 0$. Thus sabotage is more costly than working productive. Compared with the value of winning, however, k is relatively small; in particular, we assume that $k < V/18$. Player i wins the tournament if

$$x_{i0} + \sum_t e_i^t - \sum_t \sum_{j \neq i} s_{ji}^t > \max_{j \neq i} \left\{ x_{j0} + \sum_t e_j^t - \sum_t \sum_{k \neq j} s_{kj}^t \right\}, \ (k = 1, 2, 3, k \neq j)$$

where x_{i0} denotes player i's initial position. These positions are exogenously given. If a player is ahead of his competitors after both rounds, he is declared the winner. If two or more players share the leading position, each wins with equal probability. Finally, there is no discounting.

3 The second round

The model is solved by backward induction. Clearly, the players' actions in the second round depend on their relative positions. For notational convenience, we assume player 1 to be the leader after the first round, player 2 to be in second position and player 3 to be in third position (players may also be in the same position, of course). This is without loss of generality as one can simply renumber the players if necessary. Let Δ_1 denote the difference between positions of players 1 and 2, and Δ_2 the difference between players 2 and 3.

In round two, by choosing $e_i^2 = 1$ instead of $e_i^2 = 0$ a player (weakly) increases his winning-probability and so can never be worse off. Thus, by assumption A1, each player chooses $e_i^2 = 1$.

We now characterize equilibria of the subgames in round two. In some subgames, multiple equilibria exist. We do not, however, attempt to give a complete characterization of the set of all equilibria.[3] The primary objective of the paper is to highlight some potential effects of sabotage in a dynamic tournament, and to show that players' behavior in dynamic tournaments may be entirely different from behavior in static tournaments. To reach this objective, it is enough to show that certain types of equilibria exist.

Lemma 1 *Let $\Delta_1 = 0$. (i) If $\Delta_2 = 0$, there is an equilibrium, where players choose $s_{12}^2 = s_{23}^2 = s_{31}^2 = 1$. All players have a payoff of $\frac{V}{3} - k$. (ii) If $\Delta_2 = 1$, there is an equilibrium where players 1 and 2 choose $s_{12}^2 = s_{21}^2 = 1$. Player 3 chooses $s_{31}^2 = 1$ and $s_{32}^2 = 1$ with probability 0.5, respectively. Players 1 and 2 receive $\frac{V}{4} - k$, while player 3 has a payoff $\frac{V}{2} - k$. (iii) If $\Delta_2 \geq 2$, players 1 and 2 choose $s_{12}^2 = s_{21}^2 = 1$ while player 3 chooses $s_{31}^2 = s_{32}^2 = 0$. The payoffs are $\frac{V}{2} - k$ to players 1 and 2 and zero to player 3.*

Proof. (i) If any player deviates from the proposed equilibrium, he loses the tournament for sure. A deviation is therefore not profitable. (ii) If players 1 and 2 choose $s_{12}^2 = s_{21}^2 = 1$, player 3 is obviously indifferent between sabotaging either of his opponents (which he strictly prefers to not doing anything at all.) Hence, player 3 does not deviate from the proposed equilibrium. If player 1 is sabotaged by player 3, he loses for sure. Moreover, if he is not

[3] See, however, the discussion in Subsection 5.1.

sabotaged by player 3, he is indifferent between sabotaging either of his opponents. Further, as $\frac{V}{4} > k$, he prefers sabotaging one of his opponents to staying inactive. Hence, he also cannot gain by deviating from the proposed equilibrium. The same holds for player 2. It is further straightforward to show that player 3 wins with probability 0.5 so that his payoff is $\frac{V}{2} - k$ while the other players receive $\frac{V}{4} - k$. (iii) If $\Delta_1 = 0$ and $\Delta_2 \geq 2$, player 3 loses the tournament for sure and so does not sabotage. Then, players 1 and 2 obviously choose $s_{12}^2 = s_{21}^2 = 1$. As each wins the tournament with probability 0.5, they receive $\frac{V}{2} - k$. ■

Parts (i) and (iii) of this lemma are very intuitive. Part (i) says that, if all three players are at the same position, they all get the same payoff $V/3 - k$. Part (iii) says that, if one player is sufficiently far back while the other two are at the same position, then the underdog gets zero while the front-runners get a payoff of $V/2 - k$ each.[4] The most surprising part is (ii). If there are two front-runners followed by one underdog who is one step behind them, all sabotage activities are directed against the front-runners. Each of the front-runners wins with probability 1/4, while the underdog wins with probability 1/2 and is thus better off than the favorites!

The next lemma shows that being the single front-runner may be even worse. It considers the case where there is a single favorite, who is one step ahead of the second position.

Lemma 2 *Let $\Delta_1 = 1$. (i) If $\Delta_2 = 0$, there is an equilibrium, where player 1 chooses $s_{12}^2 = s_{13}^2 = 0$ and receives a payoff of zero. Players 2 and 3 choose $s_{21}^2 = s_{31}^2 = 1$. Their payoff is $\frac{V}{2} - k$, respectively. (ii) If $\Delta_2 = 1$, the players choose $s_{12}^2 = s_{21}^2 = s_{31}^2 = 1$. Each receives a payoff of $\frac{V}{3} - k$. (iii) If $\Delta_2 \geq 2$, 3 does not sabotage and has zero payoff. Player 1 chooses $s_{12}^2 = 1$ with probability $p = \frac{V-2k}{V}$ and does not sabotage with probability $1 - p$. His payoff is $V - k$. Player 2 chooses $s_{21}^2 = 1$ with probability $q = 1 - p$ and does*

[4] We use "favorite" and "front-runner" as synonyms.

not sabotage with probability $1-q=p$. *His payoff is zero.*

Proof. (i) Consider player 2. Given his opponent's actions, he is indifferent between sabotaging player 1 and 3 (which he strictly prefers to not doing anything). Hence, he does not want to deviate from the proposed equilibrium. The same holds for player 3. Then, player 1 loses for sure and does not sabotage anyone. Accordingly, players 2 and 3 receive $\frac{V}{2}-k$. (ii) If either of the players deviates from the strategy, he loses for sure. Hence, a deviation is not profitable. As all players win with equal probability, each receives $\frac{V}{3}-k$. (iii) If $\Delta_1=1$ and $\Delta_2 \geq 2$, player 3 loses the tournament for sure and so does not sabotage anybody. In order to ensure that player 1 is indifferent between choosing $s_{12}^2=1$ and $s_{12}^2=s_{13}^2=0$ we must have

$$V-k=q\frac{V}{2}+(1-q)\,V,$$

or, equivalently, $q=2k/V$. As we can easily see, player 1's payoff is $V-k$. Player 2 is indifferent between choosing $s_{21}^2=1$ and $s_{21}^2=s_{23}^2=0$ if

$$-pk+(1-p)\left(\frac{V}{2}-k\right)=0,$$

or equivalently

$$p=\frac{V-2k}{V}.$$

■

Part (i) of this Lemma 2 is on the case where one favorite is leading one step ahead of the competitors, who tie on the same position. In this case, the favorite gets sabotaged by both competitors, and thus loses for sure, while the competitors win with probability 1/2 each.

Part (ii) considers the case where the players are at adjacent positions and there are no ties. Players 2 and 3 sabotage the favorite 1, who in turn sabotages player 2. Thus all three players end up at the same position and have the same payoff.

Part (iii) is similar in spirit to part (iii) of Lemma 1. Again the distance of player 3 to the other players is too big, so player 3 essentially gives up. The leading player 1 can ensure victory by sabotaging player 2, who is initially one step behind him. Thus player 1 gets a payoff of $V - k$. The other players get zero.

We now turn to the case where player 1 is two steps ahead of player 2.

Lemma 3 *Let* $\Delta_1 = 2$. *(i) If* $\Delta_2 = 0$, *there is an equilibrium where player 1 chooses* $s_{12}^2 = 1$ *and* $s_{13}^2 = 1$ *with probability* 0.5, *respectively. Players 2 and 3 choose* $s_{21}^2 = s_{31}^2 = 1$. *Player 1's payoff is* $\frac{V}{2} - k$, *while players 2 and 3 receive* $\frac{V}{4} - k$. *(ii) If* $\Delta_2 \geq 1$, *no player sabotages. Player 1 receives* V, *while players 2 and 3 receive nothing.*

Proof. (i) It is easy to see that players 2 and 3 receive nothing, if deviating from their strategy. Moreover, by sabotaging one of his opponents, player 1 increases his winning probability from $\frac{1}{3}$ to $\frac{1}{2}$ so that his payoff is $\frac{V}{2} - k$. Obviously, each of the players 2 and 3 receives $\frac{V}{4} - k$. (ii) If $\Delta_1 = 2$ and $\Delta_2 \geq 1$, player 3 loses the tournament for sure and so does not sabotage. Accordingly, player 2 loses also for sure, while player 1 wins for sure. Thus, both these players do not sabotage, too. As player 1 wins for sure, he receives V, while his opponents receive nothing. ■

In all cases considered in Lemma 3, the favorite has a higher payoff than his opponents. Intuitively, a favorite who is far ahead of his rivals is likely to win even if his opponents direct all their sabotage activities against him.[5]

Finally, if player 1 is three ore more steps ahead of player 2, then sabotaging him makes no sense for his rivals, because player 1 will win with certainty anyhow.

[5] In case (i), there also exists an equilibrium where no player sabotages. If for example player 2 does not sabotage, it does not pay for player 3 to sabotage, too. However, this equilibrium is not coalition proof.

Lemma 4 *If $\Delta_1 \geq 3$, no player sabotages. Player 1 receives V, while players 2 and 3 receive nothing.*

Proof. If $\Delta_1 \geq 3$, players 2 and 3 lose the tournament for sure and so do not sabotage. As a best response, player 1 does not sabotage, too. As he wins for sure, he receives V, while his opponents receive nothing. ■

4 The first round

We now consider the first round. A complete characterization of the equilibrium for all possible starting differences is beyond the scope of this paper. Instead, we restrict attention to two situations which are in our view of special importance. The first situation is one where all players start from the same position, i.e. they are homogeneous ex ante. The corresponding equilibrium is described in Proposition 1.

Proposition 1 *Let $x_{10} = x_{20} = x_{30}$. Then, there is a subgame perfect equilibrium where player i chooses $e_i^1 = s_{ij}^1 = 0$. Each player's payoff (over both rounds) is $\frac{V}{3} - k$.*

Proof. There are in principle three possible deviations. If player i deviates to $e_i^1 = 1$ and $s_{ij}^1 = 0$ he receives a payoff of zero and is worse off. If player i deviates to $e_i^1 = 0$ and $s_{ij}^1 = 1$ he receives a payoff of $\frac{V}{4} - 2k$ and is worse off. If player i deviates to $e_i^1 = 1$ and $s_{ij}^1 = 1$ he receives a payoff of $\frac{V}{3} - 2k$ and is worse off. Hence, no player wants to deviate from the equilibrium. As they do not move, Lemma 2, part (i) applies and the payoffs are $\frac{V}{3} - k$. ■

Proposition 1 shows that sabotage not only has a directly destructive effect on output, but also suppresses the incentives to work productively. Note that if sabotage is not possible or too costly (for example if $k > V/3$), then in both rounds all players work productively. Thereby they tie in the end and each wins with probability 1/3. If sabotage is possible, however, no one works in the first round. The reason is that, if one player deviates and works productively, he moves ahead of his competitors. Then, in round 2 the rivals focus their sabotage activities against him. Because of this strategic effect, not working is strictly better, even though working productively has no direct cost by Assumption 1.

Our second main result characterizes a case where players are heterogeneous ex ante. The point of the proposition is to show that, in the first round, sabotage activities may be focussed against an underdog. As mentioned in the introduction, in static tournaments sabotage tends to be concentrated against favorites. We have seen that in our model this is true in the final round. In the first round, however, two favorites may focus their sabotage against one underdog.

Proposition 2 *Let $x_{10} = x_{20} = x_{30} + 1$. Then, there is a subgame perfect equilibrium where each player chooses $e_i^1 = 1$. Moreover, player 1 (2) chooses $s_{13}^1 = 1$ ($s_{23}^1 = 1$) with probability*

$$x = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{k}{2\left(\frac{V}{3} - k\right)}}$$

and does not sabotage at all with probability $1 - x$. Player 3 chooses $s_{31}^1 = 1$ with probability

$$y = \frac{k - \frac{V}{4}(1 - x)}{\left(\frac{V}{3} + k\right)(2x - 1)},$$

$s_{32}^1 = 1$ with probability y, and does not sabotage with probability $1 - 2y$.

Proof. See appendix. ■

Proposition 2 implies that the underdog, player 3, is sabotaged more heavily than the front-runners 1 and 2. The expected number of steps player 3 is moved backwards by the sabotage chosen by players 1 and 2 is $2x > 1$.

The expected number of steps that player $i = 1, 2$ is moved back by the sabotage of his rivals is y, which is smaller than 1/2 (see appendix).

5 Discussion

In this section, we discuss the robustness of our results. We do this by pointing out other possible equilibria, discussing the assumptions driving the main result, and by considering a more general model. We focus on the result from Proposition 1 that sabotage suppresses incentives to work productively in early rounds of the tournament.

5.1 Other equilibria

Above we presented one possible subgame perfect equilibrium of the game in a homogeneous and a heterogeneous setting, respectively. The game, however, has multiple equilibria. The multiplicity of equilibria is due to a coordination aspect of the game. Consider for example a symmetric subgame in stage 2. As we showed in Lemma 1, there is an equilibrium where player 1 sabotages player 2, 2 sabotages 3, and 3 sabotages 1. Of course, there is also an equilibrium where players coordinate their sabotage choices differently: 1 sabotages 3, 3 sabotages 2, and 2 sabotages 1. Moreover, there is a symmetric equilibrium where each player sabotages each rival with probability 1/2. Basically, the game has aspects of a coordination game, and there are several ways to coordinate. This is natural: sabotage against a certain player is a public good for all the other players, and of course, the private supply of a public good has features of a coordination game.

The asymmetric subgames have multiple equilibria, too, which is again due to different possible ways to coordinate. Consider for example a subgame starting with $\Delta_1 = 0$ and $\Delta_2 = 1$. As pointed out in Lemma 1, the subgame has an equilibrium where players 1 and 2 mutually sabotage each other, whereas player 3 mixes between sabotaging 1 and 2 with equal probability.

The subgame also has an equilibrium where 3 sabotages 1, 1 sabotages 2, and 2 mixes between sabotaging 1 and 3. Moreover, there is a third equilibrium which is similar to the second, with the roles of 1 and 2 exchanged. Note that in all these equilibria, for each of the front-runners 1 and 2, there is exactly one player who sabotages the front-runner with probability one. If the remaining player does not sabotage, all three players tie; thus the remaining player is indifferent between sabotaging either of his rivals. If the remaining player sabotages each of his rivals with probability 1/2, none of the players has an incentive to deviate.

The multiplicity of equilibria in the second stage also leads to a multiplicity of subgame perfect equilibria in the whole game. We highlight this by considering the case where players are homogeneous ex ante.

Remark 1 *Suppose that $x_{10} = x_{20} = x_{30}$. There exists a subgame perfect equilibrium where, in the first stage, all players work ($e_i^1 = 1$, $i = 1, 2, 3$) and do not sabotage ($s_{ij}^1 = 0$, $\forall i, j, i \neq j$).*

Proof. See appendix. ■

Remark 1 shows that in the case of homogeneous players, all players may also work productively in the first stage. As mentioned before, this is due to the multiplicity of equilibria in the second stage of the game. In the asymmetric subgames, there are also equilibria where the leading players are not sabotaged as harshly as suggested by Lemmas 1 to 3. Then, it may pay to work productively and to move one step forward in the first stage.

In our view, however, the equilibrium described in Remark 1 is less plausible than the one described in Proposition 1. If the rivals have made different choices in the first stage, we believe that it is focal to sabotage the rival who is ahead in the race. Consider player 1, for example. If player 2 is leading over 3 after the first stage, sabotaging 3 makes sense only if 1 is sufficiently certain that the sabotage decisions of 2 and 3 will compensate the advantage of 2 over 3. Moreover, given the small cost of sabotage, sabotaging 2 will be better for player 1 than not sabotaging at all, unless 1 is sufficiently sure that the choices of the other players are such that 1 wins with high probability anyhow, or 1 loses with high probability anyhow. If we require players to sabotage the rivals who are ahead in the race, the equilibrium in Remark 1 breaks down. Therefore, we have placed greater emphasis on the equilibrium described in Proposition 1.

5.2 Crucial assumptions

In this subsection we discuss the crucial assumptions behind our main result that the possibility of sabotage may lead to zero productive effort in the first stage of the game. The logic behind the result is that, in the final stage, players with a lead are sabotaged more heavily, which dampens incentives for productive work before the final stage. There are four main assumptions that drive this result.

1. We assume that there is a single winner prize. This fits very well to the example of a promotion tournament where workers often compete for a single position on a higher hierarchy level. Still, there are also real-world examples where the prize structure is different. Former General Electric CEO Jack Welch, for example, was an advocate of so-called dismissal tournaments where the worst-performing workers have to leave the firm. In such a tournament, winning means that one is allowed to stay in the firm, whereas losing implies that one gets fired. Accordingly, there are many winner and only few loser prizes. Then, in the final stage sabotage may be directed against players who are lagging behind. To understand this, consider a last place punishing scheme where all players receive an equal winner prize except the player with the lowest output. In this case, it only matters not to have the lowest output. Sabotaging front-runners may well be irrelevant, whereas sabotaging those with a low output may pay off.

2. The result hinges on there being at least three players.[6] It is easily verified that, in the model above, if there are two players, in both stages both players will work and sabotage. The crucial difference is that, with three or more players, each player can choose to sabotage a particular rival.

3. We assume that, in the first stage, players cannot get so far ahead of their rivals that they are no longer vulnerable by sabotage. To see that this is important, let the model be changed such that productive effort brings a player three or more steps ahead. Thus, if players j and k choose zero productive effort and do not sabotage in stage 1, player i can gain a lead of at least three steps, and consequently win the tournament for sure, even if both rivals sabotage him in the final stage. Then of course there is no equilibrium where players do nothing in stage 1. Conversely, if sabotage is relatively easy and effective, no player will be able to get sufficiently ahead of their rivals so that he his safe from their attack.

4. We assume that players are able to observe the ranking after the first stage of the game. Obviously, if this were not possible, players could not direct their sabotage activities against the current leader. Then, players would have an incentive to choose high productive effort in the first stage of the game.

5.3 A more general model

The result in Proposition 1 is more general than the highly stylized model considered above may suggest. In particular, it is not crucial that the number of players is exactly equal to three, that there are only two stages, or that the action spaces are discrete. In the remainder of this section, we demonstrate

[6] Chen (2003) and Münster (2007) also point out that the case of two players is different from the case of three or more players.

that a similar result can be obtained in a model with $n \geq 3$ possibly heterogeneous players, $T \geq 2$ stages, and continuous strategy spaces. The model is an extension of Münster (2007). In stage $t = 1, ..., T$, each player chooses effort $e_i^t \in \mathbb{R}_+$. Moreover, in stage $t = 1, ..., T$, each player chooses $n-1$ sabotage activities, one against each rival. Let $s_{ij}^t \in \mathbb{R}_+$ be the sabotage of i against rival $j \in \{1, ..., n\} \setminus \{i\}$ in stage t. The final output of i equals the sum of his efforts minus the sabotage received, plus a noise term ε_i that is realized after the final stage of the game has been played:

$$q_i = x_{i0} + \sum_{t=1}^{T} \left(e_i^t - \sum_{j \neq i} s_{ji}^t \right) + \varepsilon_i$$

The noise terms ε_i are identically and independently distributed according to a continuously differentiable distribution function F. As above, x_{i0} is player i 's initial position. Following Münster (2007), we assume that F has full support and is log-concave.[7] The highest output wins a winner prize $V > 0$, all others get nothing. Let p_i denote the probability that i wins. To ease notation, define

$$y_{ij} = x_{i0} + \sum_{t=1}^{T} \left(e_i^t - \sum_{l \neq i} s_{li}^t \right) - \left(x_{j0} + \sum_{t=1}^{T} \left(e_j^t - \sum_{l \neq j} s_{lj}^t \right) \right).$$

This is the difference in expected output between player i and j, given all decisions taken in the game. Then

$$\begin{aligned} p_i &= \Pr \{q_i > q_j, \forall j \neq i\} \\ &= \Pr \{y_{ij} + \varepsilon_i > \varepsilon_j, \forall j \neq i\} \\ &= \int_{-\infty}^{\infty} \left(\Pi_{j \neq i} F \left(y_{ij} + \varepsilon_i \right) \right) F' \left(\varepsilon_i \right) d\varepsilon_i. \end{aligned} \tag{1}$$

[7] The assumption of log-concavity is fulfilled by many commonly studied parametric distribution functions, see Bagnoli and Bergstrom (2005).

The objective function of player i is

$$u_i = p_i V - \sum_{t=1}^{T} \left(C_{it}\left(e_i^t\right) + S_{it}\left(\sum_{j \neq i} s_{ij}^t\right)\right).$$

Here, C_{it} and S_{it} are cost functions, assumed to be strictly increasing and convex. Following Münster (2007), we allow players to be heterogeneous and have different cost functions. In fact, if $T = 1$, the model is a special case of the model studied in Münster (2007). Moreover, we allow the cost functions to differ across stages.

To keep things as simple as possible, we assume that, for any decisions taken in earlier rounds that are not prohibitively costly, the final stage of the game has an interior pure strategy equilibrium where all decision variables are positive. This assumption implicitly limits the degree of heterogeneity between players, both concerning the exogenous differences in ability captured in the cost functions and in the initial positions, and the endogenous differences due to the play on previous stages.[8] It ensures that, at the final stage, sabotage is sufficiently easy and no player is beyond the reach of his competitors (see the third point discussed above).

As in Münster (2007, Proposition 2), in any interior equilibrium of the last stage of the game, each player will win with the same probability. The reason is easy to understand. Suppose to the contrary that player i wins with a higher probability than j. Then player k should sabotage i more and reduce his sabotage against j by the same amount. Since the cost of sabotage depends only on the sum of sabotage activities, the costs of k are unchanged. However, as shown in Münster (2007, Lemma 1), k 's probability of winning is increased; thus the initial situation cannot have been an equilibrium. Therefore, in equilibrium all players win with the same probability.

[8] As shown in Münster (2007), when players are very different, some players may not be sabotaged at all. Moreover, there has to be sufficiently much noise in order that a pure strategy equilibrium exists; this is a common feature of most models of tournaments. See Münster (2007) for a more detailed discussion of the assumption of an interior equilibrium.

It follows that, by gaining a lead in the previous stages, a player does not improve his probability of winning. Moreover, decisions taken on previous stages have no impact on the equilibrium effort and the total amount of sabotage chosen on the final stage. To see this, consider the necessary first order conditions for the optimal decisions on stage T :

$$\begin{aligned}
\frac{\partial p_i}{\partial e_i^T}V &= C_{iT}'\left(e_i^T\right) \\
\frac{\partial p_i}{\partial s_{ij}^T}V &= S_{iT}'\left(\sum_{l\neq i} s_{il}^T\right), \ \forall j \in \{1,...,n\}\setminus\{i\}
\end{aligned}$$

Using (1),

$$\begin{aligned}
\frac{\partial p_i}{\partial e_i^T} &= \int_{-\infty}^{\infty}\left(\sum_{j\neq i}\left(F'\left(y_{ij}+\varepsilon_i\right)\Pi_{l\neq i,j}F\left(y_{il}+\varepsilon_i\right)\right)\right)F'\left(\varepsilon_i\right)d\varepsilon_i, \\
\frac{\partial p_i}{\partial s_{ij}^T} &= \int_{-\infty}^{\infty}F'\left(y_{ij}+\varepsilon_i\right)\left(\Pi_{l\neq i,j}F\left(y_{il}+\varepsilon_i\right)\right)F'\left(\varepsilon_i\right)d\varepsilon_i.
\end{aligned}$$

Since every player wins with the same probability, $y_{il}=0$ for all i and $l\neq i$. It follows that

$$\begin{aligned}
\frac{\partial p_i}{\partial e_i^T} &= (n-1)\int_{-\infty}^{\infty}F'\left(\varepsilon_i\right)\left(F\left(\varepsilon_i\right)\right)^{n-2}F'\left(\varepsilon_i\right)d\varepsilon_i, \\
\frac{\partial p_i}{\partial s_{ij}^T} &= \int_{-\infty}^{\infty}F'\left(\varepsilon_i\right)\left(F\left(\varepsilon_i\right)\right)^{n-2}F'\left(\varepsilon_i\right)d\varepsilon_i.
\end{aligned}$$

Define

$$g := (n-1)\int_{-\infty}^{\infty}F'\left(\varepsilon_i\right)\left(F\left(\varepsilon_i\right)\right)^{n-2}F'\left(\varepsilon_i\right)d\varepsilon_i.$$

and note that g depends only on the fundamentals of the model, and is independent of the endogenous variables. On the last stage, equilibrium effort, and the sum of sabotage chosen by i, equals

$$\begin{aligned} e_i^T &= C_{iT}'^{-1}(gV), \\ \sum_{l \neq i} s_{il}^T &= S_{iT}'^{-1}\left(\frac{gV}{n-1}\right). \end{aligned}$$

Therefore, the costs that i incurs in the final stage are independent of the decisions taken on previous stages. Of course, the previous decisions will determine which rival i sabotages how much; only the sum of sabotage activities of i is independent of previous decisions.

We now turn to the decisions taken earlier in the game. Consider stage $T-1$. As we have just argued, the choices of player i neither change his probability of winning in the end, nor change the cost he will incur in the equilibrium of the final stage. Thus, the only impact of $i's$ choices on his payoff are through the cost i incurs in stage $T-1$. Therefore it is optimal to choose zero effort and sabotage. Proceeding with backward induction, the argument carries over to all previous stages. The following proposition sums up the discussion.

Proposition 3 *Consider the model discussed in this subsection and suppose that, for any history of the game up to the final stage T, the equilibrium of the final stage is interior and in pure strategies. Then in any subgame perfect equilibrium,*

$$e_i^t = s_{ij}^t = 0$$

for all $t \in \{1, ..., T-1\}$, all $i \in \{1, ..., n\}$, and all $j \in \{1, ..., n\} \setminus \{i\}$.

Proposition 3 shows that players choose zero effort and zero sabotage in all stages except the final stage. Thus our first main result (Proposition 1 above) is not limited to the stylized model with exactly three homogeneous players, two rounds, and discrete action spaces.

Unfortunately, we can use this more general setup only to confirm our first main result. The second result requires some players to drop out of the race and the final-stage equilibrium not to be interior. In this case, however, the general setup becomes intractable. To be able to present both results in a unified framework, we have therefore decided for the discrete model presented in Sections 2 to 4.

6 Conclusion

This paper studied sabotage in a dynamic tournament. Sabotage is not only directly harmful, but also depresses incentives to work productively, if players are homogeneous ex ante. If players are heterogeneous ex ante, sabotage may be focussed on an underdog in the first round, in contrast to findings on static tournaments. In a discussion of our results, we extended the main finding to a situation with more than two periods, where players' sabotage is not limited to one opponent. One of the main assumptions driving the result is that, for any history of the game barring prohibitively costly choices, no player is beyond the reach of the sabotage activities of his opponents. Studying situations where it is possible but costly to obtain such a lead would be an interesting extension.

A Appendix:

A.1 Proof of Proposition 2

The proof proceeds in four steps. Step (i) establishes that $x \in (1/2, 1)$ and $y \in (0, 1/2)$. Step (ii) shows that player 1 is indifferent between the two pure strategies he randomizes over, and step (iii) shows that player 1 has no incentive to deviate to any other strategy. Similar considerations apply to player 2. Finally, step (iv) shows that player 3 has no incentive to deviate.

(i) Note that

$$\frac{1}{4} > \frac{k}{2\left(\frac{V}{3} - k\right)}$$

holds iff $V > 9k$, which is true by assumption. Thus we have $1/2 < x < 1$. Next, we show that $y \in (0, 1/2)$. Note that y is positive if

$$\frac{V}{4}(1 - x) < k$$

Inserting x yields

$$\frac{V}{4}\left(\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{k}{2\left(\frac{V}{3} - k\right)}}\right) - k < 0$$

$$\Leftrightarrow \frac{V}{2} - 4k < V\sqrt{\frac{1}{4} - \frac{k}{2\left(\frac{V}{3} - k\right)}}$$

Since both sides of the inequality are positive, we can take the square of both sides. Thus the last inequality is equivalent to

$$\frac{V^2}{4} - 4kV + 16k^2 < \frac{V^2}{4} - \frac{V^2k}{2\left(\frac{V}{3} - k\right)}$$

or

$$\frac{32}{3}k^2V - \frac{8}{3}kV^2 - 32k^3 + 8k^2V < -V^2k$$

$$\Leftrightarrow -\frac{56}{3}k^2V + \frac{5}{3}kV^2 + 32k^3 > 0$$

To see that this inequality is fulfilled note that

$$\begin{aligned} -\frac{56}{3}k^2V + \frac{5}{3}kV^2 + 32k^3 &> Vk\left(-\frac{56}{3}k + \frac{5}{3}V\right) \\ &> Vk\left(-\frac{56}{3}\frac{V}{18} + \frac{5}{3}V\right) \\ &= \frac{17}{27}V^2k > 0 \end{aligned}$$

where we use $k < V/18$ in the second line. Hence $y > 0$. Moreover, y is strictly smaller than $1/2$ iff (using $x > 1/2$)

$$\frac{V(1+x)}{6} + k\,(2x-3) > 0.$$

But

$$\begin{aligned} \frac{V(1+x)}{6} + k\,(2x-3) &> \frac{V(1+\frac{1}{2})}{6} + k\,(1-3) \\ &= \frac{1}{4}V - 2k \\ &> 0. \end{aligned}$$

Therefore $y \in (0, 1/2)$.

(ii) Given the other players' strategies, player 1 is indifferent between choosing $e_1^1 = s_{13}^1 = 1$ on the one hand, and $e_1^1 = 1$ but $s_{12}^1 = s_{13}^1 = 0$ on the other, iff

$$x\left(y(V-k) + (1-2y)\left(\frac{V}{2}-k\right)\right) + (1-x)\left(2y\left(\frac{V}{3}-k\right) + (1-2y)\left(\frac{V}{2}-k\right)\right) - k$$
$$= x\left(2y\left(\frac{V}{3}-k\right) + (1-2y)\left(\frac{V}{2}-k\right)\right) + (1-x)\left(y\left(\frac{V}{2}-k\right) + (1-2y)\left(\frac{V}{4}-k\right)\right)$$

The first line is the payoff from choosing $e_1^1 = s_{13}^1 = 1$.

- With probability x, player 2 sabotages player 3. In this case:

- If 3 sabotages 2 (which happens with probability y) player 1 leads one step before player 2, and player 3 is two steps behind player 2. By Lemma 2 (iii), the payoff of 1 is $V - k$.
- If 3 does not sabotage at all (which happens with probability $1 - 2y$) then 1 and 2 tie and player 3 is three steps behind. By Lemma 1 (iii), 1 gets $\frac{V}{2} - k$.
- If 3 sabotages 1, then 1 is one step behind 2 while 3 is two steps behind 1. By Lemma 2 (iii), player 1 has a continuation payoff of zero.

- With probability $1 - x$, player 2 does not sabotage. In this case:
  - If 3 sabotages 2 (which happens with probability y) player 1 leads one step before player 2, and player 3 is one step behind player 2. By Lemma 2 (ii), the payoff to 1 is $\frac{V}{3} - k$.
  - If 3 sabotages 1 (which happens with probability y), then 1 is one step behind 2 while 3 is one step behind 1. By Lemma 2 (ii), again player 1 has a payoff of $\frac{V}{3} - k$.
  - If 3 does not sabotage at all (which happens with probability $1 - 2y$) then 1 and 2 tie and player 3 is two steps behind. By Lemma 1 (iii), 1 gets $\frac{V}{2} - k$.

Similarly, the second line is the payoff from choosing $e_1^1 = 1$ and $s_{13}^1 = s_{12}^1 = 0$.

- With probability x, player 2 sabotages player 3. This leads to the same expected continuation payoff of player 1 as in the last bullet item, except that player 1 now incurs no cost of sabotage in the first round.
- With probability $1 - x$, player 2 does not sabotage. Then:

– If 3 sabotages 2 (which happens with probability y) player 1 leads one step before player 2, who ties with 3. By Lemma 2 (i), the payoff to 1 is zero.

– If 3 sabotages 1 (which happens with probability y), then 1 and 3 tie one step behind 2. By Lemma 2 (i), player 1 gets $\frac{V}{2} - k$.

– If 3 does not sabotage at all (which happens with probability $1 - 2y$) then 1 and 2 tie and player 3 is one step behind. By Lemma 1 (ii), 1 gets $\frac{V}{4} - k$.

The equation can be transformed into

$$x\left(\frac{V}{2} - (1-y)k\right) + (1-2x)\left(\frac{V}{2} - k - \frac{yV}{3}\right) - k = (1-x)\left(\frac{V}{4} - (1-y)k\right)$$

or equivalently into

$$-x\frac{V}{4} + 2xyk + \frac{V}{4} - k - \frac{yV}{3} + \frac{2xyV}{3} - yk = 0$$

$$\Leftrightarrow y = \frac{\frac{V}{4}(1-x) - k}{\left(\frac{V}{3} + k\right)(1-2x)}$$

Thus, given the other players follow the strategies described in the proposition, player 1 is indifferent between the pure strategies over which he randomizes.

(iii) Now we show that player 1 has no incentive to deviate to any other strategy. If player 1 does not sabotage and does not work productively, he gets

$$x\left((1-2y)\left(\frac{V}{3}-k\right)+y\left(\frac{V}{4}-k\right)+y\left(\frac{V}{4}-k\right)\right)$$
$$+(1-x)\left((1-2y)\left(\frac{V}{2}-k\right)+y\left(\frac{V}{3}-k\right)+y\left(\frac{V}{3}-k\right)\right)$$

This is strictly smaller than the payoff from choosing $s_{13}^1=1$ and $e_1^1=1$ iff

$$\begin{aligned} & x\left((1-2y)\left(\frac{V}{3}-k\right)+y\left(\frac{V}{2}-2k\right)\right) \\ < \; & x\left(y(V-k)+(1-2y)\left(\frac{V}{2}-k\right)\right)-k \end{aligned}$$

Simplifying, we get

$$k<x\left(y(\frac{V}{2}+k)+(1-2y)\frac{V}{6}\right)$$

or equivalently

$$k<x\left(y(\frac{V}{6}+k)+\frac{V}{6}\right)$$

This inequality holds because, given $x>\frac{1}{2}$, the right hand side is bigger than $V/12$, and we assumed that $k<V/18$.

Now consider a deviation to $s_{13}^1=1$ and $e_1^1=0$. The expected payoff is

$$x\left((1-2y)\,0+y0+y\left(\frac{V}{2}-k\right)\right)$$
$$+(1-x)\left((1-2y)\left(\frac{V}{3}-k\right)+y\left(\frac{V}{4}-k\right)+y\left(\frac{V}{4}-k\right)\right)-k$$

Again, this is smaller than the payoff from choosing $s_{13}^1=1$ and $e_1^1=1$, because the continuation value is always smaller, independently of the realization of the mixing of the opponents in round 1.

Now consider a deviation to $s_{12}^1 = 1$ and $e_1^1 = 0$. The expected payoff is

$$
\begin{aligned}
& x\left((1-2y)\left(\frac{V}{4}-k\right)+y\left(\frac{V}{2}-k\right)+y0\right) \\
& +(1-x)\left((1-2y)\left(\frac{V}{3}-k\right)+y\left(\frac{V}{2}-k\right)+y\left(\frac{V}{4}-k\right)\right)-k \\
< \; & x\left(y(V-k)+(1-2y)\left(\frac{V}{2}-k\right)\right) \\
& +(1-x)\left(2y\left(\frac{V}{3}-k\right)+(1-2y)\left(\frac{V}{2}-k\right)\right)-k
\end{aligned}
$$

iff

$$
\begin{aligned}
0 &< x\frac{V}{4}+(1-x)\left(-y\frac{V}{12}+(1-2y)\frac{V}{6}\right) \\
&= x\frac{V}{4}+(1-x)\left(\frac{V}{6}-y\frac{5V}{12}\right)
\end{aligned}
$$

This is true since $x > 1/2$ and $y \leq 1$.

Finally, consider a deviation to $s_{12}^1 = 1$ and $e_1^1 = 1$. The expected payoff is

$$
\begin{aligned}
& x\left((1-2y)\left(\frac{V}{3}-k\right)+y\left(\frac{V}{4}-k\right)+y\left(\frac{V}{2}-k\right)\right) \\
& +(1-x)\left((1-2y)\,0+y\left(\frac{V}{3}-k\right)+y\left(\frac{V}{3}-k\right)\right)-k
\end{aligned}
$$

The continuation value is weakly smaller than that from choosing $s_{13}^1 = e_1^1 = 1$, independently of the realization of the mixing of the opponents in round 1; and for some realizations it is strictly smaller.

(iv) Consider player 3 and suppose $e_3^1 = 1$. Given the other players' strategies, player 3 is indifferent between the three alternatives to sabotage player 1, to sabotage player 2, and not sabotaging at all, if

$$2x(1-x)\left(\frac{V}{3}-k\right)+(1-x)^2\left(\frac{V}{2}-k\right)-k=(1-x)^2\left(\frac{V}{2}-k\right).$$

The left-hand-side is the payoff of sabotaging player 1, and also the payoff of sabotaging player 2; the right-hand-side is the payoff of not sabotaging at all. The equation simplifies to

$$x^2-x+\frac{k}{2\left(\frac{V}{3}-k\right)}=0$$

As $x>0$, the solution to this condition is

$$x=\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{k}{2\left(\frac{V}{3}-k\right)}}$$

It remains to consider whether player 3 wants to deviate to some other strategy. It is however straightforward to show that player 3 has no incentive to deviate.

A.2 Proof of Remark 1

Note first that if the players stick to the equilibrium strategies, they are all at the same position at the beginning of the second stage. Then, Lemma 1 applies and each player has a payoff of $\frac{V}{3}-k$.

In the first stage, there are three possible types of deviations from the proposed equilibrium. A player could deviate by choosing to work and, at the same time, to sabotage one of his opponents. Further, a player could decide not to work and not to sabotage, while thirdly, a player could decide not to work but to sabotage one of the other players. The first two deviations lead to a subgame, where two players tie in the first position, while the remaining player is one step behind. The third deviation gives rise to a subgame, where there is a single leader who is one step ahead of both his rivals.

Let us consider these subgames in more detail. As indicated before, in the first subgame where $\Delta_1 = 0$ and $\Delta_2 = 1$, there exists a different equilibrium than the one identified in Lemma 1. In particular, there is an equilibrium where all players work and player 3 sabotages player 1, player 1 sabotages player 2, whereas player 2 sabotages each of his opponents with probability 0.5. In this equilibrium, players 1 and 3 have a payoff of $\frac{V}{4} - k$, while player 2 receives $\frac{V}{2} - k$. Now consider the described first-stage deviations that lead to such a subgame. If a player deviates by choosing to work and to sabotage one of his opponents, he may fear to receive just $\frac{V}{4} - 2k$ (the payoff to player 1 minus the additional sabotage cost) which is less than he would get by sticking to the equilibrium strategy. Similarly, if a player deviates by deciding neither to work nor to sabotage, his payoff equals $\frac{V}{4} - k$ which is again less than the equilibrium payoff. Hence, depending on how the players expect the subgame to be played the first two deviations may both not be profitable.

Under the third possible deviation a player decides not to work but to sabotage one of his rivals. Here, we have $\Delta_1 = 1$ and $\Delta_2 = 0$. In addition to the equilibrium characterized in Lemma 2 there are different equilibria in this subgame. In these equilibria, all players work. Moreover, player 1 mixes between sabotaging player 2 and not sabotaging anyone (with probability $r \in \left[0, \frac{V-6k}{V}\right]$ and $1 - r$, respectively). Player 2 mixes between sabotaging player 1 and player 3 (with probability $s = \frac{V-2k}{V}$ and $1 - s$), while player 3 sabotages player 1 for sure.

To see that no player wants to deviate, note that player 1's payoff from sabotaging player 2 equals

$$(1 - s) V - k$$

whereas his payoff from not sabotaging at all is

$$(1 - s) \frac{V}{2}$$

Player 1 is indifferent between these actions if

$$(1-s)\,V - k = (1-s)\,\frac{V}{2} \Leftrightarrow s = \frac{V-2k}{V}$$

Moreover, player 1's payoff from sabotaging player 3 is

$$(1-s)\,\frac{V}{2} - k$$

which is lower than the payoffs identified before.

Similarly, player 2's payoff from sabotaging 1 or 3 is the same and given by

$$(1-r)\,\frac{V}{2} - k$$

This payoff must not be lower than the payoff from not sabotaging at all, which is

$$(1-r)\,\frac{V}{3}$$

The resulting condition is

$$(1-r)\,\frac{V}{2} - k \geq (1-r)\,\frac{V}{3}$$

or

$$r \leq \frac{V-6k}{V}$$

Finally, player 3's payoff from sabotaging 1 is

$$rsV + (1-r)\,s\frac{V}{2} - k$$

which is never lower than his payoff from sabotaging player 2

$$rs\frac{V}{2} + (1-r)\,s\frac{V}{2} - k$$

or his payoff from not sabotaging at all

$$rs\frac{V}{2} + (1-r)\,s\frac{V}{3}$$

To see the latter, note that

$$\begin{aligned}
rsV + (1-r)\,s\frac{V}{2} - k &> rs\frac{V}{2} + (1-r)\,s\frac{V}{3} \\
\Leftrightarrow s\frac{V}{6} + rs\frac{V}{3} &> k \\
\Leftrightarrow \frac{V-2k}{V}\frac{V}{6} + r\frac{V-2k}{V}\frac{V}{3} &> k \\
\Leftrightarrow \frac{V-8k}{6} + r\frac{V-2k}{3} &> 0
\end{aligned}$$

Because of $V > 8k$, the last condition is always fulfilled.

Consider now a deviation in the first stage of the game that leads to such a subgame. If a player deviates by not working and sabotaging one of his rivals, he must fear to receive a payoff $(1-r)\,\frac{V}{2} - k - k$. This payoff is lower than the equilibrium payoff if r is high enough (for $r = \frac{V-6k}{V}$ it is definitively lower). Then, the deviation is not profitable. In turn, there are situations where no profitable deviations from the proposed equilibrium exist.

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